**MARKS**

7

(6, 1)

1. The design of an electronic circuit calls for a 100-ohm resistor and a 250-ohm resistor connected in series so that their resistances add. The components from which the circuit is manufactured do not exactly meet their nominal resistances – the *actual* resistances vary independently in a way that can be modelled by normal distributions. The resistance of the 100-ohm resistors has a mean of 100 ohms and a standard deviation of 2.4 ohms, and that of the 250-ohm resistors has a mean of 250 ohms and a standard deviation of 3.2 ohms.

   (a) Find the probability the total resistance of a circuit containing one of each type of resistor selected equiprobably ('at random') and connected in series has a total resistance of between 345 and 355 ohms.

   (b) Explain briefly why it is reasonable to assume that the distributions used to model the resistances of the two resistors in the circuit can be considered to be *probabilistically independent*.

(a) Let the random variable $R_1$ represent the resistance of an equiprobably('randomly')-selected 100-ohm resistor,
and the random variable $R_2$ represent the resistance of an equiprobably('randomly')-selected 250-ohm resistor;

0.7887 (a)
Probability

the question directs us to use the models: $R_1 \sim N(100, 2.4)$ and $R_2 \sim N(250, 3.2)$ (in ohms).

Hence: $R_1 + R_2 \sim N(100+250, \sqrt{2.4^2+3.2^2}) = N(350, 4)$;
[when dealing with the **sum** of two independent normal random variables, the **distribution** remains normal, the **mean** is the sum of the two individual means, and the **standard deviation** is the square root of the sum of the squares of the two individual standard deviations].

Then:
$$\begin{aligned}\Pr(345 \le R_1 + R_2 \le 355) &= \Pr[345 \le N(350, 4) \le 355] \\ &= \Pr[-1.25 \le N(0, 1) \le 1.25] \quad \text{(standardizing)} \\ &= 2 \times (0.89435 - 0.5) \\ &= 2 \times 0.39435 \\ &= 0.7887 \simeq 79\%;\end{aligned}$$

*i.e.*, the required probability is 0.7887 or about 79%.

(b) The two types of resistors are likely manufactured **separately** from each other (*e.g.*, by different machines, perhaps in different factories).
The distributions of their resistances can therefore reasonably be modelled by **probabilistically independent** (normal) distributions.