Figure 7.1. DISCRETE PROBABILITY: Some Illustrations

Part 5 of these Course Materials introduced *continuous* probability and its mathematical development; we now turn to the corresponding ideas involving *discrete* probabilities. This introductory Figure 7.1 provides four illustrations of relevant matters, which are also discussed in the video described in Figure 7.7; the illustrations involve the following matters.

* Pubic perception of the risks (or probabilities) of some types of crime, and the suggestion that prominent reporting in the media of rare but sensational incidents systematically distorts perception (informal or involuntary personal *measuring*).
  - Underlying this article (EM9422) is the idea of also measuring *public* perception by means of *polls*; measuring has been a major theme of STAT 220 starting in Part 2, and polls are an important topic in Part 8.
* The role of risk perception in individual investor's decisions.
* 'Randomizing' a card deck by shuffling – *i.e.*, making it equally probable that any card is in any position in the deck.
* Use of the idea of *chance* in a legal context.

EM9422: The Globe and Mail, December 31, 1994, pages A1 and A9

Random violence skews crime perception

BY ALANNA MITCHELL
The Globe and Mail

Acts of random violence are the main reason for misperceptions of the amount of criminal activity in Canada and how strictly it is dealt with, the author of a federally sponsored study says.

The study for the Justice Department, conducted by University of Ottawa criminologist Julian Roberts, compared results from several years' worth of opinion polls with trends in reported crimes and victimization surveys. It is the first to look at what Canadians actually know about the criminal justice system and how they want legislators to fix it.

Because of public pressure on politicians to toughen sentences and parole eligibility, crime has become a key issue on the federal agenda and on many provincial and municipal ones.

Dr. Roberts' study, *Public Knowledge of Crime and Justice: An Inventory of Canadian Findings*, has not been publicly released but may be in the new year, according to sources in the Justice Department.

Among the findings are these:

● Although two-thirds of Canadians believe crime rates have risen during the past five years, such rates generally have remained stable.

● Most Canadians believe homicides are increasing more quickly than any other crime. In fact, the rate has remained relatively stable for 30 years and actually has begun falling, especially since the abolition of capital punishment. In 1977, the year after abolition, it was 3.06 per 100,000 population. By 1992 the rate was 2.7 per 100,000.

● Although most Canadians believe that breaking-and-entering is on the rise, this too is a misperception. In 1980 there were 26.3 such incidents reported to police for every 1,000 households. In 1990 the rate was 22.4 per 1,000 households. A recent survey conducted by Statistics Canada also showed that rates of such crimes had fallen 7 per cent between 1988 and 1993.

● The widespread view that gun use is becoming more common in crimes is also false. Thirty-seven per cent of all robberies committed in 1978 involved a firearm, but by 1990 this had fallen to 26 per cent. The proportion of homicides involving a gun has risen slightly, from 32 per cent in 1980 to 34 per cent in 1992.

● A growing proportion of Canadians (85 per cent in 1992) feel that sentences are not harsh enough. When questioned, however, they consistently underestimated the proportion of convicted criminals sent to prison.

● Canadians are highly anxious about what they perceive to be lax parole rules but, again, this is based on a misperception. They tend to believe that more than half of prisoners get parole and that more parole than ever is being granted. In fact, the federal parole-granting rate stands at 64 per cent, roughly where it has been for a decade.

● Contrary to public opinion, the majority of those paroled complete their terms in the community without committing another crime. Between 1978 and 1988, nearly three-quarters were successfully paroled. Parole was revoked for 12 per cent because they committed new crimes.

"The system is not the lenient joke people believe it to be," Dr. Roberts said yesterday from Ottawa. "Everything's not fine, but the problems are not necessarily what the public believes them to be."

Several factors have produced such an extensive list of misperceptions, he said. The most influential, though, were well-publicized acts of random violence that took place in 1994.

One was the drive-by shooting of Nicholas Battersby in Ottawa in March. Another was the shooting of Georgina Leimonis at a Toronto restaurant the next month.

Such sensational cases leave a lasting impact and heighten the public's view that danger is everywhere, Dr. Roberts said. He cited psychological literature showing that for years after a major plane crash, people wildly overestimate the risks of flying.

And because the public believes crime to be a simple phenomenon, there is tremendous support for easy answers such as harsher sentences, he said.

1 In its headline, opening sentence, and fourth-last paragraph, the article EM9422 reprinted above uses the phrase *random violence*. Look up a dictionary definition of the ordinary English meaning of *random*.
- Compare and contrast this ordinary meaning with the sense in which the word is used in the article.
- Compare and contrast this ordinary meaning with the meaning of 'random' in the phrase *simple random selecting* (which is sometimes shortened to *random selecting* and which *we* call the more evocative *equiprobable selecting*).

2 What evidence is presented in the article EM9422 reprinted above that the ... *well-publicized acts of random violence .... in 1994* were the most influential *cause* of the misperceptions?
- Explain briefly whether you consider that the inference of causality is justfied on the basis of this evidence.

(*continued overleaf*)

EM9325: The Trimark Investor, Issue 6, Spring 1993, pages 6 and 8

How risk affects individual investors

by Professor Donald Wehrung

UNDERSTANDING RISK

Do people perceive risk correctly?

Sometimes, but people perceive situations to be more risky than statistical incidence rates, when the situations are out of their control, unfamiliar, involve a high degree of media exposure (especially of catastrophic events), or involve technologies that they don't understand. For example, people perceive the chances of dying in a plane crash to be greater after extensive media coverage of a crash.

In investment settings, because individuals have different capabilities to withstand possible losses, smaller investors, facing the same market as institutional investors, perceive investments to be riskier.

Do people do a reasonable job evaluating risk?

In many cases no, because people have systematic biases in the way they assess and use probabilities. They often incorrectly estimate probabilities that are very low or very high. For example, people tend to ignore adverse, low probability events such as floods or earthquakes. They can also let their values and beliefs interfere with their risk evaluation. Wishful thinking can lead people to overestimate the chances of a big payoff. People are often too confident about their predictions, giving little credence to the possibility of extreme outcomes.

People tend to lack balance in their risk assessments. They let the "bandwagon effect" get in the way of prudent risk evaluation. When things are going well, people tend to overestimate the chance of being lucky.

As mentioned above, the media can influence people's evaluation of risk. For example, the business press often highlights investments that have done very well or very poorly. If the reported analysis is based on either short-term performance or is influenced by an extraordinary event, it may be a poor indication of future performance.

TOLERATING RISK

Do peope know their own risk tolerance?

Studies show that people often misjudge their own risk tolerance. One likely reason is that there are pressures in North American culture to see ourselves as more willing to take risks than we really are.

Can people change their risk tolerance levels?

While many believe that an individual's tolerance for risk is an unchangeable personality trait, research has shown that to be untrue.

Some stereotypes have, however, confirmed by research. For example, risk taking tends to decline with age and to increase with wealth.

Do people have the same risk tolerance for all aspects of their lives?.

Few people are either consistently risk-averse or risk-taking. Instead people take a 'portfolio' approach to the risks in their lives. In other words, people have an overall aggregate level of tolerable risk. As the risks increase in one area of life (*e.g.*, a marriage breakup), acceptable risks in other areas decrease. Before considering what level of investment risk is acceptable, it is important for people to know what other risks they're facing.

CONTROLLING RISK

Do people have to accept risk as it is?

Not at all. Controlling uncertain events is an effective way of changing their riskiness. People reduce investment risk by seeking professional advice and management; educating themselves; diversifying and monitoring their investments.

SUMMARY

While risks provide both excitement and stimulation, most people prefer to avoid unnecessary or foolhardy risks. When assessing your tolerance for investment risk, you should evaluate the overall levels of risk in all aspects of your life. If you already have high risks in areas such as career, health, or family, it may not be wise to take on additional high financial risks, or vice versa. Remember the old adage "Do you want to eat well or sleep well?"

Risk is pervasive and will always be with us. The best we can do is to foresee some of the risks ahead and to control or mitigate some of the outcomes.

Donald A. Wehrung is Associate Dean and Professor of Management Science and Policy Analysis, Faculty of Commerce and Business Administration, University of British Columbia. Over the past twenty years he has studied, along with his colleague Kenneth MacCrimmon, the risk behaviour of many North American executives. These executives answered an extensive portfolio of questionnaires and participated in interviews about hypothetical risky situations, how they handled naturally-occurring risky situations, and their attitudes toward risk. These studies, many of which involved personal investment decisions, have revealed important insights regarding the investment behaviour of senior managers in particular and of investors more generally. Detailed discussion of the findings is available in Taking Risks: The Management of Uncertainty (by *K.R. MacCrimmon and D.A. Wehrung*, The Free Press, paperback, 1988*).*

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3 The article EM9325 reprinted above and EM9422 overleaf on page 7.3 on public perception of crime incidence levels make some of the same, and some different, points. Compare and contrast the two articles with respect to their discussion of *risk*.

4 Explain in probabilistic terms what you understand by the last sentences of the third and the fourth paragraphs in the left-hand column of the article EM9325 reprinted above:

- *People are often too confident about their predictions, giving little credence to the possibility of extreme outcomes*;
- *When things are going well, people tend to overestimate the chance of being lucky.*

5 The *Living Webster Encyclopedic Dictionary of the English Language* (Chicago, 1974, page 829) gives the ordinary meaning of the noun *risk* as: *exposure to the chance of injury or loss; a hazard or dangerous chance.* Describe briefly what you understand to be the *relationship*, if any, between the concepts of *risk* and *probability.*

(*continued*)

Figure 7.1. DISCRETE PROBABILITY: Some Illustrations (continued 1)

EM9015: The Globe and Mail, January 27, 1990, page D4

Seven shuffles are needed to put gamble in card deck

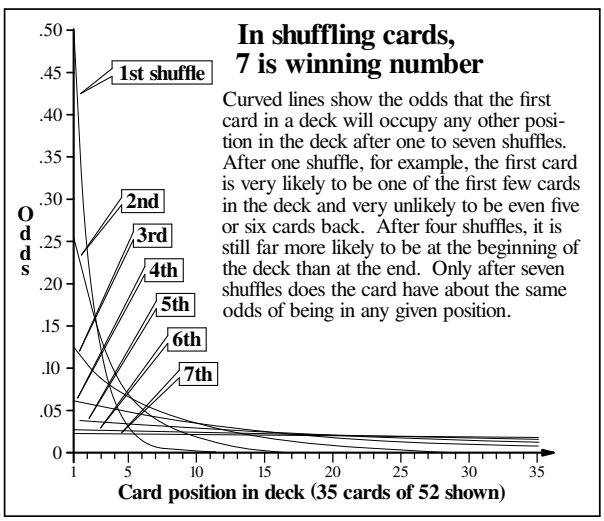
BY GINA KOLATA
New York Times Service

IT TAKES SEVEN ordinary, imperfect shuffles to mix a deck of cards thoroughly, researchers have found. Fewer are not enough and more do not significantly improve the mixing.

The mathematical proof, discovered after studies of results from elaborate computer calculations and careful observation of card games, confirms the intuition of many gamblers, bridge enthusiasts and casual players.

The finding has implications for everyone who plays cards and everyone, from casino operators to magicians, who has a stake in knowing whether a shuffle is random.

The mathematical problem was complicated because of the immense number of possible ways the cards in a deck can be arranged; any of 52 could be first in the deck, any of 51 could be second, 50 could be third and so on. Multiplied out, the number of possible permutations is 10 with 62 zeros after it.



N.Y. Times News Service

No one expected that the shuffling problem would have a simple answer, said Dave Bayer, a mathematician and computer scientist at Columbia University who is a co-author of the recent discovery. Other problems in statistics, like analyzing speech patterns to identify speakers, might be amenable to similar approaches, he said.

The new result "definitely solves the problem," said David Aldous, a statistician at the University of California at Berkeley. "All their calculations are right. It's a fascinating result."

Persi Diaconis, a mathematician and statistician at Harvard University who is the other author of the discovery, said the methods used are already helping mathematicians analyze problems in abstract mathematics that have nothing to do with shuffling or with any known real-world phenomenon.

Dr. Diaconis, who is also a magician, has invented numerous card tricks and has been carefully watching casino dealers and casual card players shuffle for the past 20 years. The usual shuffling produces a card order that "is far from random," he said. "Most people shuffle cards three or four times. Five times is considered excessive."

The realization that most shuffled decks are not actually random allows gamblers to improve their odds of winning. "There are people who go to casinos and make money on this," he said. "I know people who are out there doing that now."

In Las Vegas, cards are shuffled from four to seven times, at the discretion of the casino owners, said Richard Ingram, a Las Vegas enforcement agent for the Nevada gambling control board.

Dr. Diaconis said he almost never sees a dealer shuffle seven times.

He said his research also shows that when dealers shuffle several decks at once, they need to shuffle more. Two decks should be shuffled nine times, he said, and six decks should be shuffled 12 times, which is unheard of in the casinos.

At Trump Plaza in Atlantic City, N.J., blackjack dealers shuffle eight decks twice at the beginning of each game, said Howard Dreitzer, who is senior vice-president of casino operations. "We've tested these shuffles and feel that they are random," he said, adding that "no one has ever complained."

Bridge players usually shuffle about four times, except in some tournaments where a computer randomly mixes the cards, said Edgar Kaplan, who is editor and publisher of Bridge World magazine.

Asked whether he expected bridge players to change their shuffling habits, he replied, "There will be a few who will be affected and will doggedly shuffle seven times to the irritation of everyone else."

6 What definition or description is given in the article EM9015 reprinted above of what constitutes *shuffling*?
- What is the *consequence* of your answer for the information conveyed by the article? Explain briefly.

7 The second paragraph of the article EM9015 uses the phrase .... *mathematical proof*; comment briefly on the meaning of the word 'proof' in this phrase in light of the information given in this second paragraph (and elsewhere in the article).

8 Using ideas on page 7.23 (the first side of Figure 7.5), show how the number 10^{62} at the end of the fourth paragraph of the article EM9015 reprinted above is obtained.
- If you have access to a suitable calculator, obtain the value of this number of permutations to 4 significant figures; is your (correct) value different enough from the value in the article to meaningfully affect the discussion? Explain briefly.
  - A normal deck of cards contains four suits (spades, hearts, diamonds, clubs) each having 13 cards that are often taken as having face values from 1 to 11 (or 13). Because many cards games are more concerned with face value than suit, explain briefly whether the (enormous) number of permutations of 52 cards is the *only* number relevant to the discussion.

(*continued overleaf*)

9 In the left-hand column of the article EM9015 reprinted overleaf on page 7.5, starting in the second paragraph there are multiple references to the disciplines of mathematics, computer science and statistics; *e.g.*, *mathematical proof, elaborate computer calculations, mathematical problem, mathematician and computer scientist, statistician, mathematician and statistician*. What discipline relevant to the discussion in the article is *not* mentioned?

- Explain briefly why framimg the discussion in these terms might mislead a reader with limited knowledge of the disciplines of mathematics, statistics and computer science.

EM9203: The Globe and Mail, January 8, 1992, page A7

Court cuts drunk-driving sentences

Canadian Press

VANCOUVER – The difference between a drunk driver who kills someone and one who doesn't is mostly chance, the B.C. Court of Appeal ruled in reducing jail terms for three impaired drivers who killed or maimed others.

"The moral culpability of an offence is determined by the state of mind which accompanies the offender's unlawful act," Mr. Justice Josiah Wood said in his written reasons for judgement.

"The moral culpability of the offence of impaired driving ... is the same as that of impaired driving causing either death or bodily harm."

"The fact death or bodily harm does or does not result ... is more likely to be due to chance than to any circumstance of foreseeability."

The rulings were handed down unanimously by a panel consisting of Chief Justice Alan McEachern and Justices Henry Hutcheon, John Lambert, Sam Toy and Judge Wood.

John Grezenda, 45, of Victoria had an eight-year sentence reduced to four years, David McNeil, 27, and John Sweeney, 23, had their sentences of 4½ years cut to 18 months. The court upheld the eight-year sentence of Curtis Lunn, 47.

The article EM9203 reprinted at the right is of interest in this Figure 7.1 because of its concern with 'chance'.

The article implies that, in the Canadian legal system, 'moral culpability' is determined by the nature of a crime, *not* by its consequences. Thus:

- the sentence for murder should not depend on whether the victim is, say, a philanthropist who leaves behind numerous family members and friends and whose donations have enriched the lives of many people, or is someone without family or dependents and who has no public profile; OR:
- the sentence for arson should not depend on whether, say, the building destroyed is an irreplaceable heritage structure with priceless contents or an abandoned shell whose high market value is due only to the location of its site.

The article EM9203 reprinted at the right also implies, in its fourth-paragraph contrast of *chance* with *foreseeability*, that the latter refers to the individual case and *not* to behaviour under repetition; however, the criminal code is likely concerned with impaired drivers because data for behaviour *under repetition* indicate (foresee?) that such drivers have a *higher probability* of causing death or bodily harm than *non*-impaired drivers.

A related matter of statistical interest is how to *measure* driving impairment; for example, see the article EM0202: *Drawing the line on drinking and driving.*

10 Subject to the (limited) information in the article EM9203 reprinted at the right, give a succinct description of the meaning of 'chance' as the word is used in the article's fourth (and first) paragraphs.

- Compare and contrast this meaning of 'chance' with that of 'odds' in, for instance, the last sentence of the comment in the diagram at the upper right overleaf on page 7.5: *Only after seven shuffles does the card have about the same odds of being in any given position*.
- What should a reader infer from the qualifiers of *chance* of 'mostly' (in the third line of the article) and 'more likely' (in the second line of the fourth paragraph)? Explain briefly.

11 To the extent feasible from a relatively short article, set out the steps in the 'reasons the judgement' that the justices appear to have used to justify reducing three sentences (but *not* a fourth) mentioned in the last paragraph of the article EM9203. Comment critically on your answer.

The four articles EM9422, EM9325, EM9014 and EM9203 reprinted in this Figure 7.1 are also used in Statistical Highlight #40.