Figure 2.5b. SAMPLE ERROR: Investigating Coin Weights

The problem of getting good measurements and finding ways to describe them concisely has been our chief concern in the preceding chapters. Only indirectly has there been any suggestion that there is sometimes a problem of picking representative items to be subjected to measurement. In Chapter 4, to determine the homogeneity of the magnesium alloy, we tested 50 spots on five bars chosen from 100 bars. Was this a fair sample? We omitted any reference to the question of whether the 923 gasoline pumps could be considered an adequate sample of the total collection of pumps in the country. Taking a large number of items does not guarantee getting a representative selection of the whole supply or population. Many investigators have found this out the hard way.

Before tackling the general problem of how to get a good selection of items for measurement, let us investigate a special case. Your author went to the bank and got two rolls of newly minted, 1984 or later, copper-zinc alloy pennies. For the moment, let us accept without argument that these 100 coins give an adequate picture of the coins minted during the work period in which they were made. Undoubtedly the coins accumulated in a big tray and got mixed up in the process. There is a specified weight for cents, together with legal tolerances for minimum and maximum weight of a coin. The nominal weight in September 1984 was 2.51 grams, with a permitted tolerance of 0.1 grams above and below the nominal weight.

The problem was to determine whether or not this sample of 100 pennies fell within the permitted tolerance of 2.51 ± 0.1 grams. To do this investigating your author had access to a very fine balance that could be read to the *millionth* part of a gram. Such precision was quite unnecessary. The weights were recorded to tenths of a milligram; that is, to the fourth place of decimals. Even that was really unnecessary. Weights to the nearest milligram would have been quite good enough. Why?

The actual weights of the coins vary over a range of about 250 milligrams. Weighing to the nearest milligram would surely be good enough since the weighing error could, at most, extend the range of actual weights by only a milligram or two in 250 milligrams. The weights are correct to four places of decimals. How do we know that? If we reweighed the 100 coins using any other fine balance capable of weighing accurately to six decimal places, we would get exactly the same weights over again out to four decimal places. The possibility of a constant error common to all weights was eliminated by checking the balance with a standard weight.

All the above detail is directed to establish that the observed scatter of the weights is not a result of errors in weighing. The measurement error is nil for this investigating. An effort to determine whether the weights of the coins do or do not conform to specification must depend on examining a number of coins. For this purpose we need a balance good enough that a coin will not be called outside the tolerance limit because our weighing introduces additional scatter into the results. There is no measurement error in the weights as recorded. The variation among the weights found is a property of the coins, and in no way reflects measurement error.

The individual weights are given in class intervals of 20 milligrams for each roll of coins. These are displayed in Table 2.5c.1 in Figure 2.5c. A glance at the totals in the last column suggests that the actual weights of the coins are distributed among the class intervals in very much the same manner as are the measurement errors on one object such as the paper thickness measurements.

When the coins themselves were arranged in columns corresponding to the class intervals, they formed a histogram that is indistinguishable from that in Figure 2.7b. We have here not 100 crude measurements on one object, but one very careful measurement on each of 100 objects. Nevertheless, the 100 results are distributed in the same form as the normal law of error.

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We delayed answering the question as to whether these 100 coins are completely representative of the population of newly minted cents. The chances are very good, of course, that the coins are representative of some rather short interval in the total annual production of cents. If we desire to report on the cents minted in any year, we should plan to put aside a few coins from each day's work. These coins should be thoroughly mixed and a sufficient number drawn from the collection. The coins are enough alike that there is no danger of bias or intentional selecting, either for those going into the collection or for those drawn for weighing.

Coins are easy to set aside and mix thoroughly. However, there are many cases in which it is physically impossible to do this. For example, bales of raw wool weigh several hundred kilograms, and there may be hundreds in a shipment. Custom duties are assessed on the basis of the percent of clean wool in the shipment. As the bales vary considerably, it becomes necessary to sample the shipment by taking two or more cores from each of a number of bales. Economy of effort requires that no more bales be sampled than is necessary to obtain a satisfactory estimate of the percent of clean wool in the entire shipment. We have already learned one way to sample fairly. The bales may be marked 1 to **N**. Then **N** cards are numbered 1 to **N**. If n bales must be sampled, n cards are drawn from the carefully shuffled pack.

1 Near the end of the first paragraph, Youden states: *Taking a large number of items does not guarantee getting a representative selection of the whole .... population*. What *are* the advantage(s), if any, of a large sample?
- What are the *dis*advantage(s)?

2 Explain why Youden mentions that the coins .... *got mixed up* .... in the discussion in the second paragraph.

3 Youden states in the third paragraph that the balance he used to weigh the coins .... *could be read to the* millionth *part of a gram*. Explain briefly what this statement implies about the balance's:
- accuracy;
- precision.

4 Explain the *meaning* of the statement in the fourth paragraph: *The weights are correct to four places of decimals.*
- Is this statement *true*? Explain briefly.

(*continued overleaf*)

5 Explain briefly why Youden gave the results *separately* in Table 2.5c.1 in Figure 2.5c on page 2.43 for the two rolls of 50 coins, instead of giving only the last column for all 100 coins?

6 Explain why, in the seventh paragraph, Youden describes the histograms in Figures 2.5c and 2.7b as *indistinguishable*, when this does not *appear* to be the case.

7 In the eighth paragraph, Youden states: .... *we should plan to put aside a few coins from each day's work*. Explain briefly the reason for this suggestion.

8 In the last sentence of the eighth paragraph, Youden states that *selecting bias* is not a problem when sampling the coins.

- Are there sampling situations where selecting inaccuracy *is* a concern?
  - if *Yes*, describe two cases;
  - if *No*, explain why.

9 Youden states in the last paragraph: *We have already learned one way to sample fairly.* Explain the meaning and significance of the word *fairly* in this context.

10 Youden describes two *sampling* situations – coins and bales of wool.

- Which is the more *difficult* unit to sample?
- Describe the *factors* which make the unit you choose the more difficult of the two.
  - Which of the two situations (coins or bales of wool) will lead to *less* precise estimate(s)? Explain briefly.

11 How did Youden address the possible *inaccuracy* of the coin weights?

- Did this procedure *eliminate* measuring inaccuracy in the weights? Explain briefly.
- Explain briefly whether this procedure would *always* be applicable for dealing with measuring inaccuracy.

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