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MEASURING: Paper Thickness

In scientific work the knack of estimating tenths of a division on scales and instrument dials becomes almost automatic. The way to acquire this ability is to get some practice. We will now undertake an investigation that will quickly reveal your skill in reading subdivisions of a scale interval. The investigation that we are to undertake is to measure the thickness of the paper used in one of your textbooks. Although a single sheet of paper is much thinner than the smallest scale divisions on a ruler, a simple procedure will make it possible for you to determine the thickness of the paper quite precisely. The procedure consists of reading the thickness of a stack of sheets of the paper and dividing the reading by the number of sheets in the stack. Simple as this procedure appears, we will find that it reveals quite a lot about measuring.

Prepare a form for recording the measurements. Strangely enough, careful measurements are sometimes so poorly recorded even by professional scientists that not even the experimenter can interpret them sometime later. A form suitable for this experiment with spaces to enter four separate observations is shown in Table HL39.1 below. Note that the form identifies the investigator, the source of the data, and the date of the experiment or observation. These are characteristics of useful forms.

Choose a book without inserts or special paper. First open the book near the beginning and also near the end. Hold the stack of pages together. This is the stack whose thickness we will measure with a ruler marked off in centimetres and millimetres. We will estimate tenths of a millimetre.

There are a number of details to observe even in this simple investigation. Read the numbers on the first page of the stack and that on the page facing the last page of the stack. Both numbers should be odd. The difference between these two numbers is equal to the number of pages in the stack. This difference must always be an even number. Each separate sheet accounts for two pages, so divide the number of pages by two to get the number of sheets. Enter these data on the record form.

Pinch the stack firmly between thumb and fingers and lay the scale across the edge of the stack. Measure the thickness of the stack and record the reading. The stack will usually be between one and two centimetres thick; *i.e.*, between 10 and 20 mm (millimetres). Try to estimate tenths of a millimetre. If this seems too hard at first, at least estimate to the nearest one-fourth (0.25) of a millimetre. Record these readings as decimals. For example, record 14 and an estimated one-fourth of a division as 14.25.

Table HL39.1

Example of a form for keeping a record of measurements
Experiment to measure thickness of sheets of paper

Investigator: B. G. Date: March 16, 1981

observation number	page number at front	page number at back	pages in stack	sheets in stack	stack thickness (mm)	thickness per sheet (mm)
1	67	387	320	160	12.4	0.0775
2	41	459	418	209	16.4	0.0785
3	23	549	526	263	20.0	0.0760
4	35	521	486	243	18.5	0.0761
					Total	0.3081
					Average	0.0770

Measurements Do Not Always Agree

After you have made the first measurement, close the book. Then reopen it at a new place and record the new data. Make at least four measurements. Now divide the reading of the thickness by the number of sheets in the stack. The quotient gives the thickness of one sheet of paper as a decimal part of a millimetre. When this division is made for each measurement, you will certainly find small differences among the quotients. You have discovered for yourself that measurements made on the *same* thing do not agree perfectly. To be sure, the number of sheets was changed from one measurement to the next. But that does not explain the disagreement in the answers. Certainly a stack of 200 sheets should be just twice as thick as a stack of 100 sheets. When the stack thickness is divided by the number of sheets we should always get the thickness of a single sheet.

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It may be argued that all four answers in Table HL39.1 agree in the first decimal place. Clearly, all the answers are a little less than 0.08 mm. Thus any one of the results rounded off would give us this answer. Why take four readings?

Just to take a practical everyday reason, consider the paper business. Although paper in bulk is sold by weight, most users are also concerned with the thickness of paper and size of the sheet. Thick paper will mean fewer sheets of a given size per unit weight paid for. A variation of as little as 0.01 mm – the difference between 0.08 and 0.09 mm – would reduce the number of sheets by more than ten percent. We need to know the answer to one or two more deci- mal places. In a situation like this, it is usual to obtain the average of several readings. You should note, however, that although repetition alone doesn't ensure accuracy, it does help us locate errors.

Many people seem to feel that there is some magic in the repetition of measurements and that if a measurement is repeated frequently enough the final result will approach a "true" value. This is what scientists mean by accuracy.

1. Carry out the procedure for measuring paper thickness using a course text (*e.g.*, Moore and McCabe); set out your observations as in Table HL39.1 at the left above.

2. Give reasons for the differences among your four measured values of paper thickness.
   - Suggest appropriate ways in which the differences among your values could be reduced in magnitude.
   - Can you see any *common* characteristic(s) of your suggestions?

3. What is the reason for using *different* front and back page numbers in the four repetitions of the measuring process?

4. Why is it a good idea to make the number of pages in the stack as *large* as reasonably possible?
   - What factor(s) place a limit on how large this number can be made?

(*continued overleaf*)

5 If you were to measure the thickness of the stack with a *more precise* measuring instrument than a ruler, what would be the effect on the differences among the measured values of paper thickness?

- How could the differences be made to *disappear*?

6 Youden says in the second last paragraph overleaf on page HL39.1: .... *repetition alone doesn't ensure accuracy*, (but) *it does help us locate errors*. Describe any *other* reasons for taking repeated measurements.

- When is it *not* possible to take repeated measurements? Give specific examples of such situations.
- Explain whether repeated measurements are usually taken in sample surveys like opinion polls and political polls.
  - Briefly describe the consequences of your answer for the *accuracy* of poll results.

NOTE: A set of paper thickness measurements made by Youden's students is given in Figure 3.8 of the STAT 231 Course Materials.

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