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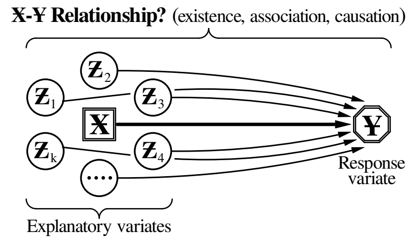
‘CONFOUNDING’: Useage in Statistics

1. Variety of Useage: Four facets of ‘confounding’



As background to understanding confounding when answering Question(s) about an X-Y relationship between a focal variate X and a response variate Y, Z_1, Z_2, ..., Z_k in the schema at the right are called **lurking variates**, a phrase that means lurking *explanatory* variates in that each Z accounts, at least in part, for changes from element to element in the value of the response variate. The importance of lurking variates is that, if the distributions of their values *differ* between groups of elements [like (sub)populations or samples] with different values of the focal variate, an Answer about the X-Y relationship may differ from the true state of affairs unless the differences in the values of the relevant Zs are taken into account. *Our* definition of ‘confounding’ is:

∗ **Confounding:** differing distributions of values of one or more *non*-focal explanatory variate(s) among two (or more) groups of elements or units [like (sub)populations or samples] with different values of the focal variate.

Dictionary meanings of ‘confounding’ in ordinary English include *confused*, *bewildered* and *mixed up* – the last of these three is closest to our statistical meaning given above, because the effects on Y of differences in X *and* in one or more of the Zs are ‘mixed up’ (or ‘cannot be separated’ as it is also expressed) – see also the discussion of the confounding effect on page HL3.3 of this Highlight #3. The *difficulty* with the statistical useage is that different statisticians in different places may, without distinction, use ‘confounding’ to refer to any one of four of its facets:

- the **definition:** inability (or failure) to separate the effects of X and Z_i [or X_i and X_j] (which are *associated*) on Y,
- the **idea:** non-focal explanatory (or lurking) variate(s) Z_i *differ in value* for different X values,
- the **limitation:** an Answer to a Question about an X-Y relationship that may be meaningfully different from the ‘truth’,
- the **consequence:** an Answer may be *altered* in a meaningful (*i.e.*, practically important) way if the values of (one or more) Z_i are taken into account.

This variety of useage, reflecting lack of agreement among statisticians about how broadly ‘confounding’ is to be interpreted, can obscure its underlying *idea*, which is the facet emphasized in, for example, our introductory discussion in Section 2 on page HL57.2 in Statistical Highlight #57; it can also be a source of confusion. [There is, of course, common ground among the four facets (most obviously among the last three) because they all refer to the *same* phenomenon.]

2. Some Distinctions: Four types of confounding

As summarized in Table HL3.1 below, one way to make these matters more transparent is to distinguish four contexts for ‘confounding’ in statistics; to do so in these Statistical Highlights, *we* qualify ‘confounding’ with one of four adjectives:

● perfect, ● partial, ● general, ● selecting.

However, these adjectives and distinctions are particular to these Statistical Highlights (and Course Materials) and are unlikely to be encountered or understood elsewhere – this is like *our* use of ‘EPS from an unstratified population’ instead of the usual SRS’ (see Note 1 on page HL85.3 in Statistical Highlight #85) and of ‘EPA’ instead of ‘randomization’ (see the lower half of page HL10.6 in Highlight #10). The latter three facets of confounding are encompassed by our definition of comparison error (from page HL57.2 in Statistical Highlight #57).

Table HL3.1: SUMMARY OF USAGE OF ‘CONFOUNDING’ IN STATISTICS
(Simpson’s Paradox referred to below is discussed in the previous Figure 11.6 pages 11.11 to 11.18)

Description	Type	.... Impact		Facet	Illustration
Perfect confounding	1	Positive:	Exploited in DOE	Definition	Fractional factorial treatment structure
Partial confounding	2	Negative:	Imposes	Idea, limitation	‘Confounding’ in comparative Plans
General confounding	3		limitation on	Limitation, consequence	‘Confounding’ *and* Simpson’s Paradox
Selecting confounding	4		an Answer	Consequence, limitation	Judgement selecting

∗ **Comparison error:** for an Answer about an X-Y relationship that is based on comparing attributes of groups of elements or units with different values of the focal variate(s), comparison error is the difference from the *intended* (or *true*) state of affairs arising from:
– differing distributions of lurking variate values between (or among) the groups of elements or units OR **–** confounding.

More details about the four facets of confounding and our distinctions are as follows:

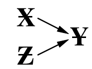
○ Confounding (‘perfect or type 1 confounding’) is a term in the statistical area of *Design of Experiments* (DOE), where it indicates inability to (fully) separate the effects of two (or more) ***focal*** variates on a response variate; it can be exploited to achieve statistical benefits in Plans with a *fractional* factorial treatment structure – see Note 5 on page HL68.2 in Statistical Highlight #68.

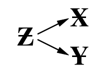
- The adjective *perfect* for type 1 confounding indicates that levels of (some) *focal* variates and/or their interactions are associated with correlation of magnitude 1 – this is why (some of) the effects on a response variate *cannot* be separated (except by using a Plan with a *full* factorial treatment structure).
  - \+ A Plan with a *fractional* factorial treatment structure accepts the limitation on Answers imposed by (‘perfect or type 1’) confounding to obtain the advantage of using *fewer* resources resulting from a *smaller* number of runs – in this sense, confounding of *focal* variates [introduced by the investigator(s)] in DOE has a *positive* impact.
- This is likely the original useage of ‘confounding’ in statistics – the emphasis in this useage is on the (original) *definition.*

(*continued overleaf*)

+ When 'confounding' (in the sense of its definition) is introduced among two or more ***focal*** variates and/or their interactions in a fractional factorial treatment structure, this facet is *not* encompassed by our definition of comparison error. However, 'partial or type 2 confounding' among *non*-focal lurking variates *is* a potential source of *our* comparison error.

○ Confounding ('partial or type 2 confounding') in the context of comparative Plans imposes a limitation on an Answer to a Question with a causative aspect, due to one (or more) [*non*-focal] confounders changing (or differing) as the focal variate changes (or differs) in value. The impact of type 2 confounding is *negative* and the emphasis is on the *idea* of confounding and the resulting *limitation* imposed on Answers by comparison error.

(8) 

(9) 

– We distinguish two cases of type 2 confounding – either may give rise to comparison error that distorts reality (creates illusion) and so leads to a 'wrong' Answer about an **X**-**Y** relationship:
  + when **Z** and **X** *both* cause **Y** (type 2a) – this situation is that of our introduction to confounding in Section 2 of Statistical Highlight #57, and the relevant causal structure, from Statistical Highlight #59, is case (8) [equivalent to case (1) with the confounder shown explicitly] shown again at the right;
  + when **Z** is a *common cause* of **X** and **Y** (type 2b) – the relevant causal structure is case (9) at the right above [so-called **common response**] and see also Statistical Highlight #60 and Appendix 2 on the facing page HL3.3 and page HL3.4.

– The adjective *partial* for type 2 confounding indicates that the association of [the (*un*wanted) change in] the confounder **Z** and (the change in) the focal variate **X** has a correlation that is (usually) *less* than 1 in magnitude;
  + The special case of *zero* correlation is discussed briefly near the bottom of page HL58.2 in Statistical Highlight #58.

– In the 2004 STAT 231 Course Notes, 'confounding' means our 'partial confounding'.

○ Confounding ('general or type 3 confounding') is a broader meaning used by some statisticians to encompass both the 'partial' confounding of comparative Plans *and* the effects of lurking variates in phenomena like Simpson's Paradox. The impact of type 3 confounding is (again) *negative* and the emphasis is on the *limitation* on Answers and its *consequence*.

– The adjective *general* is to remind us an Answer [usually to a Question about a (*causal*) **X**-**Y** relationship] may be *altered* in a meaningful (*i.e.*, a practically important) way if the values of **Z** are taken into account.

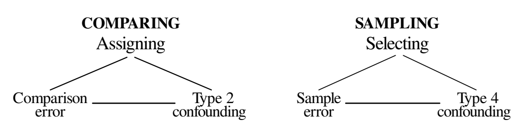
– When phenomena like Simpson's Paradox are considered to be an instance of ('general or type 3') confounding, discussion of its management (in an observational Plan) in Section 7 on pages HL51.4 to HL51.6 in Statistical Highlight #51 supplements earlier discussion of managing confounding (*e.g.*, see Table HL6.1 on page HL6.5 in Statistical Highlight #6).

– Simpson's Paradox and related phenomena (discussed in Statistical Highlight #51 would not usually be considered to involve *causation* in the sense of the discussion of Statistical Highlight #61. As a consequence, inclusion of Simpson's Paradox in 'general or type 3 confounding' affects the wording (or implication) of two definitions:
  ∗ **Causative aspect:** the Answer from the investigation of a **causative** Question addresses some characteristic(s) of a *relationship* between a response variate and one (or more) explanatory variates; if the relationship is *causal*, the intent is usually that *changing* the value(s) of the explanatory variate(s) would (or will) change the response variate value.
  ∗ **Focal variate:** an explanatory variate whose *relationship* to the response variate is involved in the Answer to the Question.

  If Simpson's Paradox and related phenomena are *not* regarded as instances of 'confounding', a causative aspect and the focal variate would both be defined (or considered) as involving a *causal* relationship (*e.g.*, see pages HL91.5 and HL91.9 in Statistical Highlight #91) and our distinction involving 'general or type 3 confounding' would not be needed.

○ Confounding ('selecting or type 4 confounding') involves the possible *creation* of an *unwanted* relationship (*e.g.*, by judgement selecting) between unit sample inclusion probabilities and response variate values – see Statistical Highlight #83. The *relationship* here is between $\mathbf{X}^*$ [which indicates whether a unit *is* selected for the sample ($\mathbf{X}^*$= 1) or is in the group of units *not* selected ($\mathbf{X}^*$= 0)] and **Y**, distinct from the *Question* which may have a descriptive *or* a causative aspect.

– Type 4 confounding is unique to these Statistical Highlights (and Course Materials) and is included in this Highlight #3 primarily to provide statistical insight from recognizing common themes of probability *assigning* and probability *selecting*;
  + probability assigning (*e.g.*, EPA) manages type 2 confounding,
  + probability selecting (*e.g.*, EPS) manages type 4 confounding;



  'manages' here means 'provides a basis for statistical theory that quantifies the likely magnitude of (comparison or sample) error' – this theory shows that both processes are more likely to achieve their goal of acceptable limitation on an Answer with *in*creasing group or sample size(s).

  Type 2 confounding (both cases) *distorts* a (wanted) relationship; type 4 confounding *creates* an *un*wanted relationship.

– The impact of type 4 confounding is (again) *negative* and the emphasis is on the *consequence* (and *limitation*).

NOTES: 1. A further difficulty with 'confounding' is that its root may be used in any of three forms; we can say, for example:
- there is *confounding* of the effects of variates **X** and **Z** on variate **Y**, OR:
- the effects of variates **X** and **Z** on variate **Y** are *confounded*; ALSO:
- if **X** is the focal variate, then **Z** (which is associated with **X**) is a possible *confounder*.

2. The *association* (*e.g.*, non-zero correlation) of confounded variates is really only an *incidental* feature of the phenomenon – association in the *usual* state of affairs for variates that change together.

(*continued*)

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'CONFOUNDING': Useage in Statistics (continued 1)

NOTES: (cont.) 2. ● *Zero* correlation of confounded variates is usually introduced by the investigator(s) – for instance, in a factorial treatment structure [see the brief discussion of diagram (5) just before Note 1 on page HL58.2 in Highlight #58].

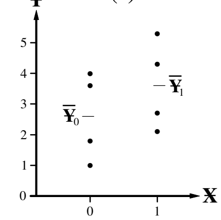
3. Key ideas to take from the (lengthy) discussion of confounding in this Highlight #3 are:
- the use and meaning of 'confounding' in DOE,
- the idea and management of 'confounding' (or 'lurking variates') in comparative Plans, taking into account the *two* ways a change in a lurking variate can affect attribute value(s) [see also Section 3 on page HL51.3 in Highlight #51]:
  - by *causing* elements' response variate (and, hence, their attribute) values to change, AND:
  - by distorting attribute *calculation* when subdividing is used to manage comparison error in an observational Plan.

For an introductory statistics course, whether Simpson's Paradox and related phenomena are instances of 'confounding' is of no consequence and the concept of 'selecting or type 4 confounding' is *solely* for enrichment.

Surprisingly, 'confounding' may not be mentioned elsewhere in discussion of statistical methods – for instance, it does not appear in the index (p. 500) of the widely-cited text by G.W. Snedecor and W.G. Cochran, *Statistical Methods*, The Iowa State University Press, Ames, Iowa, Seventh Edition, 1980.
- No mention of 'confounding' may indicate its interpretation in this text as solely our 'perfect or type 1 confounding' (the 'original' definition), coupled with no *formal* discussion of the topic of DOE by Snedecor and Cochran.

3. Appendix 1 – The Confounding Effect

(2)



In an *observational* Plan, for a focal variate with q values, we think of the respondent population as being made up of q *sub*populations; each subpopulation is those elements which have a particular value of the focal variate. Diagram (2) at the right shows an instance of q = 2 with the two subpopulations being of the *same* size (4 elements); two short horizontal lines show the two subpopulation average responses $\overline{Y}_0$ and $\overline{Y}_1$ [see also diagram (1) at the centre right of page HL10.3 in Highlight #10]. The difference between $\overline{Y}_1$ and $\overline{Y}_0$ for the two sub*populations* has two components:
- ∗ the *treatment effect* arising from their different X values;
- ∗ an effect due to differences between the two subpopulations in the distributions of values (*e.g.*, in the averages) of one or more lurking variates – *we* call this the **confounding effect** and we write equation (HL3.1) below;

$\overline{Y}_1 - \overline{Y}_0$ = effect of change in X + effect of change in Z_1, ..., Z_k = treatment effect + confounding effect. -----(HL3.1)

Explanatory variates are usually numerous and so, for each element, as these variates take their 'natural' values *un*influenced by the investigator(s), there is ample opportunity for different distributions of one or more Z_i among the q subpopulations of the respondent population. It is usually feasible to manage at most a *few* Zs by matching and/or subdividing.

Assessing Answers from observational Plans must take account of the confounding effect because:
- it is a source of comparison error and the resulting limitation imposed on the Answer(s),
- the treatment effect and the confounding effect cannot be quantified *separately* – we can only know their sum;

thus, our efforts to manage an *inherent* limitation on Answers from observational Plans meet, at best, with only *partial* success.
For further discussion and illustration of the confounding effect, see pages HL9.5 and HL9.6 in Section 6 of Statistical Highlight #9.

4. Appendix 2: Connections Among Three Variates

To provide statistical perspective on confounding, we recognize that for *three* variates [two expanatory (X and Z) and one response (Y)] involving *two* causal relationships, there are *five* causal structures, as shown in the first two columns of Table HL3.2 at the right below; the first structure has *two* contexts, making six lines in the Table. The structures are five of the twelve cases given in Statistical Highlight #59 on page HL59.1 [cases (4), (6), (8), (9) and (10)] plus case $(1)_2$ from the centre right of page HL63.1 in Highlight #63 [see also the second bullet (●) of Note 7 on page HL60.4 in Highlight #60]. [A reminder of the definition of 'interaction' is:
- ∗ **Interaction** of two factors X_1 and X_2 is said to occur when the effect of one factor on a response variate Y depends on the level of the other factor. Interaction means the combined effect of two factors is *not* the sum of their individual effects.]

Several matters are noteworthy.
- ○ What tends to distinguish the cases is the pattern of the *causal* relationships in the last column of Table HL3.2 – as shown in the *third* column, each variate is *associated* with each of the other two, except in the case of interaction, when the X-Z relationship is not relevant.

Table HL3.2

	Variate *causal* connections	Name	Association X-Y	Association X-Z	Association Z-Y	Causation X-Y	Causation X-Z	Causation Z-Y
(8)	$Z \searrow$, $X \rightarrow Y$ ≡ $X, Z \rightarrow Y$	Confounding (type 2a) [A common response Y]	Yes	Yes	Yes	Yes	No	Yes
$(1)_2$	$Z \searrow$, $X \rightarrow Y$ ≡ $X_1, X_2 \rightarrow Y$	Interaction	Yes	---	Yes	Yes	No	Yes
(9)	$X \swarrow Z \searrow Y$ ≡ $Z \rightarrow X, Y$	Confounding (type 2b) [A common cause Z]	Yes	Yes	Yes	No	Yes	Yes
(10)	$\nearrow Z$, $X \rightarrow Y$ ≡ $X \rightarrow Z, Y$	A common cause X	Yes	Yes	Yes	Yes	Yes	No
(6)	$X \nearrow Z \searrow Y$ ≡ $X \rightarrow Z \rightarrow Y$	Causal chain XZY	Yes	Yes	Yes	Yes	Yes	Yes
(4)	$\swarrow Z$, $X \rightarrow Y$ ≡ $Z \rightarrow X \rightarrow Y$	Causal chain ZXY	Yes	Yes	Yes	Yes	Yes	Yes

(*continued overleaf*)

- ○ *Confounding* and *interaction* have similarities (**+**) and differences (**–**).
  - **+** both involve two explanatory variates which cause a response variate;
  - **+** both have the same pattern of causal relationships and (except as noted overleaf on page HL3.3) associations in Table HL3.2;
  - **–** their focus is different:
    - ⊙ confounding is concerned with the impact on investigating the $\mathbf{X}$-$\mathbf{Y}$ relationship of (unwanted) *changes* in (confounder) $\mathbf{Z}$ as (focal variate) $\mathbf{X}$ changes;
    - ⊙ interaction is concerned with the impact on the $\mathbf{X}_1$-$\mathbf{Y}$ relationship *and* the $\mathbf{X}_2$-$\mathbf{Y}$ relationship of the *value* of (focal variates) $\mathbf{X}_2$ and $\mathbf{X}_1$ respectively (see also Notes 1 and 2 on page HL65.2 in Statistical Highlight #65).

    The same components but different focus of confounding and interaction are somewhat reminiscent of the conditioning-ignoring distinction discussed in Note 3 in Highlight #60 because, in each case, statistical *mis*handling provides opportunity for comparison error to impose unnecessary (and so, possibly unacceptable) limitation on Answers.

  The *manifestation* of confounding as comparison error may be (adversely) affected *if* there is *interaction* of $\mathbf{Z}$ and $\mathbf{X}$ – for example, (puzzling) 'inconsistencies' may be exhibited in the $\mathbf{X}$-$\mathbf{Y}$ relationship – see Table HL60.1 and the discussion on the respective lower and upper halves of pages HL60.3 and HL60.4 in Statistical Highlight #60.
- ○ The possibility of $\mathbf{Z}$ as a *common cause* of $\mathbf{X}$ and $\mathbf{Y}$ [case (9)] is relevant when establishing the reason for an $\mathbf{X}$-$\mathbf{Y}$ association, as discussed on pages HL60.3 and HL60.4 in Statistical Highlight #60.
  - **–** Common cause $\mathbf{Z}$ responsible for *mis*identifying $\mathbf{X}$ as a cause of $\mathbf{Y}$ is our type 2b confounding – recall the discussion near the top of page HL3.2 and see the bottom of page HL60.3 and the top half of page HL60.4 in Statistical Highlight #60.
    - **+** From *this* perspective, $\mathbf{Z}$ as a common cause of $\mathbf{X}$ and $\mathbf{Y}$ could be regarded as an extreme case of our type 2a confounding where $\mathbf{Z}$ is *solely* responsible for the change in $\mathbf{Y}$ as $\mathbf{X}$ changes.
- ○ $\mathbf{X}$ as a *common cause* of $\mathbf{Z}$ and $\mathbf{Y}$ [case (10)] is really the causal structure at the right, because $\mathbf{Z}$ is an *explanatory* variate; thus, case (10) really involves *three* causal relationships, *not* two as in this Highlight #3, but we see in Statistical Highlights #64 and #10 that case (10) with *three* causal relationships:

(10) $$\begin{tikzcd} & \mathbf{Z} \arrow[dr] & \\ \mathbf{X} \arrow[ur] \arrow[rr] & & \mathbf{Y} \end{tikzcd}$$

  - **–** is *not* a viable basis for a comparative Plan, as discussed in Note 2 on page HL64.3 in Highlight #64;
  - **–** can result in *biased* estimating of a treatment effect, as illustrated on pages HL10.3 and HL10.4 in the discussion of Table HL10.4.
- ○ As discussed in Statistical Highlights #61 and #62 (in the middle of page HL61.1 and in Note 1 near the middle of page HL62.1), we think of causation of $\mathbf{Y}$ by $\mathbf{X}$ as proceeding via a (long) **causal chain** of explanatory variates leading to the response of interest. The Question context identifies (arbitrarily) *one* (focal) variate ($\mathbf{X}$) in this chain as being of interest, but we recognize that this variate is *preceded* by and followed by other 'focal' variates; the context also (arbitrarily) defines the *end* of the chain in terms of a particular *response* variate ($\mathbf{Y}$). However, this response can become part of an *explan- atory* variate chain if a different Question context identifes a *different* (later) response variate. From this perspective:
  - **–** The causal chain of case (6) is merely the upper branch of the (real) causal structure of case (10) shown above at the right;
    - ⊙ case (6) reminds us to distinguish $\mathbf{X}$ causing $\mathbf{Y}$ *via* $\mathbf{Z}$ from $\mathbf{X}$ and $\mathbf{Z}$ as *separate* causes of $\mathbf{Y}$ [case (8)].
  - **–** The causal chain of case (4) is really case (1) [= case (8)] – $\mathbf{Z}$ in case (4) is merely an explanatory variate *preceding* the focal variate $\mathbf{X}$ in the causal chain and so is (generally) of no statistical interest in the Question context.

The (surprising) number of statistical issues arising with relationships among *only three* variates is further complicated if the $\mathbf{X}$-$\mathbf{Z}$-$\mathbf{Y}$ relationship is modelled mathematically; such a model (for use in the Analysis stage of the FDEAC cycle) needs to consider:

- ● the *form* in the model (*e.g.*, first power, second power, square root, logarithm, product) of $\mathbf{X}$ and $\mathbf{Z}$;
- ● the *distribution* of $\mathbf{Z}$ (*e.g.*, its mean and standard deviation) [and perhaps of $\mathbf{X}$];
- ● the *relationship* of $\mathbf{X}$ and $\mathbf{Z}$ (*e.g.*, their correlation).

Association of (focal variate) $\mathbf{X}$ and (confounder) $\mathbf{Z}$ in case (8) is one feature of confounding, a source of comparison error and limitation on Answers from comparative Plans. Similarly, association among variates in the structural component (on the right-hand side) of a response model [like equation (HL57.1) on page HL57.1 in Statistical Highlight 57] is also a source of such limitation, manifested as *uncertainty* in the estimates of model parameters [*e.g.*, β_1 – the treatment effect for $\mathbf{X}$ – in equation (HL57.1)].

- ○ This uncertainty becomes apparent from *stepwise* model fitting, a process to assess [*e.g.*, based on the coefficient of multiple determination, a measure of the proportion of the variation in $\mathbf{Y}$ accounted for by the fitted model] *which* explanatory variates to include in the model. For instance, in the case of two (focal) variates $\mathbf{X}_1$ and $\mathbf{X}_2$, three models are fitted – one with *both* variates, one with $\mathbf{X}_1$ only and one with $\mathbf{X}_2$ only. The stronger the association (in the data) of $\mathbf{X}_1$ and $\mathbf{X}_2$, the greater the likely difference in the estimates of their coefficients β_1 and β_2 among the three models.
  - **–** In the extreme situation where two variates $\mathbf{X}_i$ and $\mathbf{X}_j$ have correlation of magnitude 1 (*i.e.*, $\mathbf{X}_i$ and $\mathbf{X}_j$ are the *same* variate statistically), the model-fitting process with these *two* variates cannot be achieved computationally – the design matrix is not of full rank and so cannot be inverted.

In introductory statistics courses, emphasis on comparative Plans with *one* focal variate, together with similarities of confounding and interaction when there are three variates, should not be allowed to obscure the continuing importance of possible confounding in comparative Plans with *two or more* focal variates. With *three* variates and possible confounders $\mathbf{Z}_i$, $\mathbf{Z}_j$ and $\mathbf{Z}_k$, statistical issues like those in the foregoing discussion may arise for connections among:

○ $\mathbf{X}_1$, $\mathbf{Z}_i$ and $\mathbf{Y}$, ○ $\mathbf{X}_2$, $\mathbf{Z}_j$ and $\mathbf{Y}$, AND ○ $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{Z}_k$ and $\mathbf{Y}$ [$\mathbf{Z}_k$ may be a *common cause* of $\mathbf{X}_1$, $\mathbf{X}_2$ *and* $\mathbf{Y}$].