1. **Problem**

Let $\Phi : M_d \to M_r$ be the map defined by $\Phi(X) = \frac{Tr(X)}{r} I_r$. Prove that this map is CPTP, find a Choi-Kraus representation of $\Phi$, and find the operator system $S_{\Phi}$ of $\Phi$.

2. **Problem**

Let $\Psi : M_d \to M_d$ be defined by

$$\Psi(X) = \frac{Tr(X)I_d + X}{d + 1}.$$ 

Prove that this map is CPTP, find a Choi-Kraus representation of $\Psi$, find $\alpha(\Psi)$.

3. **Problem**

Let $\Psi : M_d \to M_d$ be defined by

$$\Psi(X) = \frac{Tr(X)I_d - X^t}{d - 1}.$$ 

Prove that this map is CPTP and find $\alpha(\Psi)$.

4. **Problem**

Let $A \in M_d$ be a positive semidefinite matrix. Note that $S_A$ is CPTP if and only if the diagonal entries of $A$ are all 1’s. Prove that $\alpha(S_A) = d$.

5. **Problem**

Let $A = \begin{pmatrix} 1 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1 & 0 \\ 1/\sqrt{2} & 0 & 1 \end{pmatrix}$. Prove that $A$ is positive semidefinite and find the dimension of the largest Knill-LaFlamme protected subspace of $S_A$. 


6. Problem

Let $\sigma_n = (\mathbb{Z}_n, +)$ be the cyclic group of order $n$, let $u = u_1 \in \mathbb{C}(\sigma_n)$ be the generator, and let $\omega = e^{2\pi i/n}$ be the primitive $n$-th root of unity. Verify the claim from the notes that if we set

$$e_j = 1/n \sum_{k=1}^{n} (\bar{\omega}^j u)^k,$$

where all additions and products are in the $*$-algebra $\mathbb{C}(\sigma_n)$, then $e_j = e_j^* = e_j^2$ and $u = \sum_{j=1}^{n} \omega^j e_j$.

7. Problem

Consider the CHSH game with probability on the inputs defined by

$$\pi((0, 0)) = \pi((0, 1)) = \pi((1, 0)) = 1/6, \ \pi((1, 1)) = 1/2.$$ 

Find an optimal deterministic strategy for this game and prove that it is optimal.

Can you find a quantum strategy that does better?

8. Problem

Let $G$ be a graph. Prove that the $k$-colouring game for $G$ has a perfect deterministic strategy if and only if $k \geq \chi(G)$. 

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