QIC890/PMATH950
HOMEWORK SET 4
DUE APRIL 3, 2020

VERN I. PAULSEN

1. Problem

Let $\Phi : M_d \to M_r$ be the map defined by $\Phi(X) = \frac{Tr(X)}{r} I_r$. Prove that this map is CPTP, find a Choi-Kraus representation of Φ, and find the operator system $\mathcal{S}_\Phi$ of Φ.

2. Problem

Let $\Psi : M_d \to M_d$ be defined by

$$\Psi(X) = \frac{Tr(X)I_d + X}{d+1}.$$

Prove that this map is CPTP, find a Choi-Kraus representation of Ψ, find $\alpha(\Psi)$.

3. Problem

Let $\Psi : M_d \to M_d$ be defined by

$$\Psi(X) = \frac{Tr(X)I_d - X^t}{d-1}.$$

Prove that this map is CPTP and find $\alpha(\Psi)$.

4. Problem

Let $A \in M_d$ be a positive semidefinite matrix. Note that S_A is CPTP if and only if the diagonal entries of A are all 1's. Prove that $\alpha(S_A) = d$.

5. Problem

Let $A = \begin{pmatrix} 1 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1 & 0 \\ 1/\sqrt{2} & 0 & 1 \end{pmatrix}$. Prove that A is positive semidefinite and find the dimension of the largest Knill-LaFlamme protected subspace of S_A.

6. Problem

Let $\sigma_n = (\mathbb{Z}_n, +)$ be the cyclic group of order n, let $u = u_1 \in \mathbb{C}(\sigma_n)$ be the generator, and let $\omega = e^{2\pi i/n}$ be the primitive n-th root of unity. Verify the claim from the notes that if we set

$$e_j = 1/n \sum_{k=1}^{n} \left(\bar{\omega}^j u\right)^k,$$

where all additions and products are in the *-algebra $\mathbb{C}(\sigma_n)$, then $e_j = e_j^2 = e_j^*$ and $u = \sum_{j=1}^n \omega^j e_j$.

7. Problem

Consider the CHSH game with probability on the inputs defined by

$$\pi((0,0)) = \pi((0,1)) = \pi((1,0)) = 1/6, \ \pi((1,1)) = 1/2.$$

Find an optimal deterministic strategy for this game and prove that it is optimal.

Can you find a quantum strategy that does better?

8. Problem

Let G be a graph. Prove that the k-colouring game for G has a perfect deterministic strategy if and only if $k \geq \chi(G)$.

Institute for Quantum Computing and Department of Pure Mathematics, University of Waterloo, Waterloo, ON, Canada N2L 3G1

E-mail address: vpaulsen@uwaterloo.ca