SYDE Advanced Math 2, Assignment 5 Due Fri July 12, 11:00 pm

1: Find the Fourier series for the 2π-periodic function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x$ when $-\pi \leq x < \pi$, then use Parseval's Identity to show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

2: Let $f : \mathbb{R} \to \mathbb{R}$ be the function of period 4 given by $f(x) = 1$ for $-1 \leq x < 1$ and $f(x) = 0$ for $1 \leq x < 3$. Find the Fourier series for f, then evaluate at $x = 0$ to find $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.

3: Let $f : \mathbb{R} \to \mathbb{R}$ be the 2π-periodic function with $f(x) = x^3 - \pi^2 x$ for $-\pi \leq x \leq \pi$. Find the Fourier series for f, then evaluate at $x = \frac{\pi}{2}$ to find $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3}$.

4: Use Fourier series to solve the ODE $4x'' + x = f(t)$, for $x = x(t)$, where $f : \mathbb{R} \to \mathbb{R}$ is the 2π-periodic function given by $f(t) = t^2$ for $-\pi \leq t \leq \pi$.