PMATH 367 Topology, Assignment 2.5 Not to be handed in

1: For each of the following subsets $A \subseteq \mathbb{R}^n$, determine whether A is closed, whether A is compact, and whether A is connected.

(a) $A = \left\{ (a,b,c,d) \in \mathbb{R}^4 \,\middle|\, \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)^2 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right) \right\}$.

(b) A is the set of points $(a,b,c) \in \mathbb{R}^3$ such that the polynomial $p(x) = x^3 + ax^2 + bx + c$ has three distinct real roots which all lie in the closed interval $[-1,1]$.

2: (a) Let $A \subseteq \mathbb{R}^2$. Show that if A is countable then $A^c = \mathbb{R}^2 \setminus A$ is path-connected.

(b) Let $I = [0,1] \subseteq \mathbb{R}$. Find the path-components of $X = I^2$ using the dictionary order. topology