PMATH 347 Groups and Rings, Assignment 7 Not to hand in

1: Find all Sylow subgroups, and all normal subgroups, of S_4.

2: (a) Prove that there is no simple group G with $|G| = 56$.

(b) Show that every group of order 66 is isomorphic to one of the groups $\mathbb{Z}_{66}$, $\mathbb{Z}_{11} \times D_3$, $\mathbb{Z}_3 \times D_{11}$ or D_{33}.

3: (a) List all irreducible polynomials of degree 1, 2 and 3 in $\mathbb{Z}_2[x]$, and determine the number of irreducible polynomials of degree 4 in $\mathbb{Z}_2[x]$.

(b) Let $p \in \mathbb{Z}^+$ be an odd prime number. Find the number of irreducible monic cubic polynomials in $\mathbb{Z}_p[x]$.

4: (a) Determine which of the following polynomials $f(x)$ are irreducible in $\mathbb{Q}[x]$.

(i) $f(x) = \frac{5}{2}x^5 + \frac{9}{2}x^4 + 15x^3 + \frac{3}{7}x^2 + 6x + \frac{3}{14}$.

(ii) $f(x) = 55x^5 + 21x^2 + 45$

(iii) $f(x) = x^4 + x^3 + 3x^2 + 2x + 2$

(b) Factor each of the following polynomials $f(x)$ into irreducible factors in $\mathbb{Q}[x]$, in $\mathbb{R}[x]$ and in $\mathbb{C}[x]$.

(i) $f(x) = 15x^4 - 2x^3 + 4x^2 + 11x + 2$

(ii) $f(x) = 3x^5 - x^4 - 6x^3 + 2x^2 - 6x + 2$.