In this assignment, T_u denotes the translation by the vector u, $R_{p,\theta}$ denotes the rotation about the point p by the angle θ, F_L denotes the reflection in the line L and $G_{u,L}$ denotes the glide reflection $G_{u,L} = T_u F_L = F_L T_u$ where u is parallel to L.

1: (a) Let $a = (1,8)$, $b = (2,1)$ and $c = (3,4)$. Find the image of the triangle abc under $G_{(-2,-6),y=3x-8} R_{(3,8),90^\circ}$.

(b) Let L be the line $2x - 3y = 1$. Find the equation of the line M such that $F_M F_L = T_{(-2,3)}$.

(c) Let L be the line $2x - 3y = 1$. Find the equation of the line N such that $F_N F_L = R_{(2,1),90^\circ}$.

2: (a) Express the isometry $R_{(4,4),90^\circ} F_{x+3y=6}$ as a glide reflection.

(b) Express the isometry $F_{y=3x} T_{(-2,3)} G_{(2,1),x+2=2y}$ as a single rotation.

3: (a) Find a set $X \subset \mathbb{R}^2$ with $|X| = 4$ such that $\mathrm{Sym}(X) = \{I, F_{x+2y=5}\}$.

(b) Find a set $Y \subset \mathbb{R}^2$ with $|Y| = 4$ such that $\mathrm{Sym}(Y) = \{I, R_{(1,2),\pi}\}$.

(c) Find $\mathrm{Sym}(Z)$, where $Z = \{(x,y) \in \mathbb{R}^2 \,|\, y^2 = x^2 - x^4\}$.

(d) Let $X = \{(x,y,z) \in \mathbb{R}^3 \,|\, xyz = 0\}$ and let G be the rotation group of X. Determine whether the rotation group of X is isomorphic to $\mathbb{Z}_n$, D_n, A_4, A_5 or to S_4.

4: (a) How many 9-bead necklaces (up to D_9 symmetry) can be made using 3 colours?

(b) How many ways (up to rotational symmetry) can the 12 vertices of a regular icosahedron be coloured using 2 colours?