PMATH 336 Intro to Group Theory, Assignment 5 Due Tues July 16, 11:00 pm

1: (a) For each of the two quotient groups $U_{16}/\langle 7\rangle$ and $U_{16}/\langle 9\rangle$, list all elements in each coset, determine the multiplication tables, and determine whether the group is isomorphic to $\mathbb{Z}_4$ or to $\mathbb{Z}_2\oplus\mathbb{Z}_2$.

(b) List all the elements in each left coset of $H=\langle F_0\rangle$ in D_4 and find all the conjugate subgroups, that is find all subgroups of D_4 of the form aHa^{-1} for some $a\in D_4$.

2: (a) Let $H=\big\{(1),(134),(143),(13),(14),(34)\big\}\leq S_4$. List all of the elements in each left coset of H in S_4 and determine whether $H\trianglelefteq S_4$.

(b) Let $N=\{(1),(12)(34),(13)(24),(14)(23)\}\leq S_4$. Show that $N\trianglelefteq S_4$ and determine whether S_4/N is isomorphic to $\mathbb{Z}_6$ or to D_3.

3: (a) (The Orbit/Stabilizer Theorem) Let A be a nonempty set and let G be a finite subgroup of $\mathrm{Perm}(A)$. For $a\in A$, the **orbit** of a is the set $\mathrm{Orb}(a)=\big\{\sigma(a)\,\big|\,\sigma\in G\big\}\subseteq A$, and the **stabilizer** of a is the set $\mathrm{Stab}(a)=\big\{\sigma\in G\,\big|\,\sigma(a)=a\big\}$. Show that for all $a\in A$, we have $\mathrm{Stab}(a)\leq G$ and $|G|=\big|\mathrm{Orb}(a)\big|\,\big|\mathrm{Stab}(a)\big|$.

(b) Let $G=\big\{(1),(13)(46),(13)(25),(14)(36),(16)(34),(25)(46),(1436)(25),(1634)(25)\big\}\leq\mathrm{Perm}\{1,2,\cdots,n\}$. Find $\mathrm{Orb}(1)$, $\mathrm{Stab}(1)$, $\mathrm{Orb}(2)$ and $\mathrm{Stab}(2)$.

(c) Let $G=GL_3(\mathbb{Z}_2)\leq\mathrm{Perm}({\mathbb{Z}_2}^3)$ and let $a=e_1=(1,0,0)^T\in{\mathbb{Z}_2}^3$. Find $\big|\mathrm{Orb}(a)\big|$ and $\big|\mathrm{Stab}(a)\big|$.

4: (a) Find all the homomorphisms $\phi:\mathbb{Z}_4\to\mathbb{C}^*$. For each one, describe its kernel and image.

(b) Show that $\mathbb{S}^1/C_n\cong\mathbb{S}^1$ where $\mathbb{S}^1=\big\{z\in\mathbb{C}^*\big||z|=1\big\}$ and $C_n=\big\{z\in\mathbb{C}^*\big|z^n=1\big\}$.

(c) Let H be the spiral $H=\big\{r\,e^{i\,\theta}\in\mathbb{C}^*\big|r=e^{\theta/\pi}\big\}\leq\mathbb{C}^*$. Show that $\mathbb{C}^*/H\cong\mathbb{S}^1$.