Name (print): _____

Signature: _____

ID Number: _____

SYDE 311, Advanced Engineering Math 2
Midterm Test, Spring Term, 2024
University of Waterloo

Instructor: Stephen New

Date: Wednesday July 3rd, 2024

Time: 4:30-6:20 pm

Place: EXP 1689

Instructions:

1. Place your name, signature and ID number in the spaces provided at the top of this page.

2. This test contains 6 pages, including this cover page and a page at the end for extra space.

3. No calculators or any other electronic devices are allowed.

4. Answer all 4 questions; all questions will be given equal value.
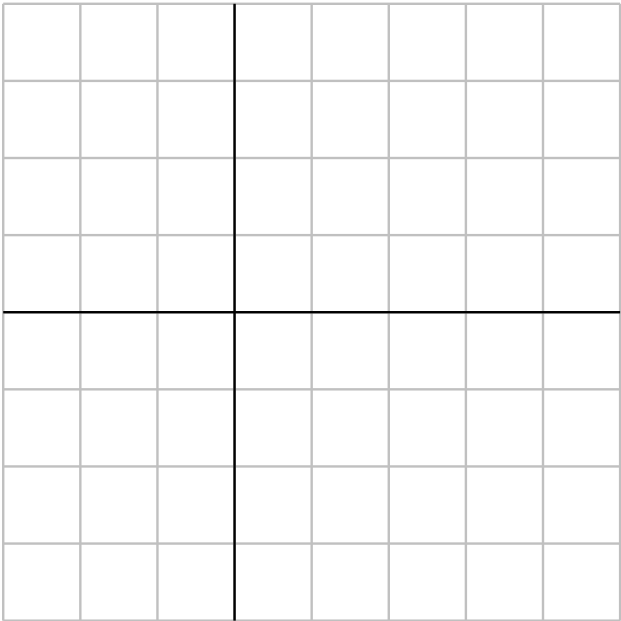
5. Provide full explanations with all your solutions.

Question	Mark
1	/10
2	/10
3	/10
4	/10
Total	/40

[10]   **1:** Consider the ODE given by $x^2 y'' - 2xy' + 2y = x + 2$ for $y = y(x)$ with $x > 0$.

(a) Use variation of parameters to solve the given ODE given that $y = y_1(x) = x$ and $y = y_2(x) = x^2$ are solutions to the associated homogeneous ODE $x^2 y'' - 2xy' + 2y = 0$.

(b) Solve the IVP given by $x^2 y'' - 2xy' + 2y = x + 2$ with $y(1) = 2$ and $y'(1) = 3$.

[10] **2:** Consider the pair of ODEs given by $x' = y^2$ and $y' = x - 1$.

(a) Sketch a phase portrait for this ODE: include the isoclines $x' = 0$, $y' = 0$ and $\frac{y'}{x'} = c$ for $c = \pm\frac{1}{4}, \pm 1, \pm 4$, and the direction field, and the solution curves through $(0, b)$ for $b = 0, \pm 2$.

(b) Find a conserved quantity $H(x, y)$ for this pair of ODEs, and use it to find the exact values of $f(2)$ and $f(4)$ where $y = f(x)$ is the solution curve through $(0, 0)$.

[10]   **3:** Consider the pair of ODEs given by $x' = y - x - 2$ and $y' = x^2 - y$.

(a) Find all the equilibrium points.

(b) For each equilibrium point, determine whether it is an attracting point, a repelling point, or a saddle point.

(c) Find the solution to the linearized system at each of the equilibrium points.

[10] **4:** Solve the ODE $(2+x^2)y'' + 4xy' + 2y = 0$ using power series centred at 0. Find an explicit, closed form formula for the general solution.

Use this page to continue solutions if you require additional space. If you do this, then clearly indicate which questions are continued here.