SYDE Advanced Math 2, Assignment 3 Not to hand in

1: (a) The substitution $u(x) = y'(x)$ and $u'(x) = y''(x)$ transforms a second order DE of the form $y'' = F(y', x)$ for $y = y(x)$ to the first order DE $u' = F(u, x)$ for $u = u(x)$. Use this substitution to solve the IVP $xy'' + y' = 1$ with $y(1) = 2$ and $y'(1) = 3$.

(b) The substitution $u(y(x)) = y'(x)$ and $u'(y(x))y'(x) = y''(x)$ transforms a second order DE of the form $y'' = F(y', y)$ for $y = y(x)$ to the first order DE $u\,u' = F(u, y)$ for $u = u(y)$. Use this substitution to solve $y\,y'' + (y')^2 = 0$ with $y(1) = 2$ and $y'(1) = 3$.

2: Consider the IVP $y'' = y\,y'$ with $y(0) = 1$ and $y'(0) = 1$.

(a) Find the exact solution $y = f(x)$ to the given IVP.

(b) With a calculator, use Euler's method with step size $\Delta x = 0.2$ to approximate $f(1)$.

3: Solve the following IVPs.

(a) $y'' + 3y' + 2y = 0$ with $y(0) = 1$, $y'(0) = 0$

(b) $y'' + 4y' + 5y = 0$ with $y(0) = 3$, $y'(0) = 1$

(c) $4y'' - 4y' + y = 0$ with $y(1) = 1$, $y'(1) = 2$

4: Solve the following linear ODEs.

(a) $y'' - 2y' + 5y = 10x^2 - 3x$

(b) $y'' + 2y' - 2y = 3xe^{2x}$

5: Solve the following linear ODEs.

(a) $2y'' + y' - y + x + e^{-x} = 0$

(b) $y'' - 6y' + 10y = e^{3x} \sin x$

6: Solve the following IVPs.

(a) $4y'' - y = x$ with $y(0) = 2$, $y'(0) = 1$

(b) $y'' - 6y' + 9y = e^{3x}$ with $y(0) = 1$, $y'(0) = 0$

7: Solve the following third-order linear ODEs.

(a) $y''' + 2y'' - 5y' - 6y = 0$

(b) $y''' - 3y' + 2y = 2 \sin x$