

**1:** Consider the IVP  $y' = x - y^2$  with  $y(0) = 0$ .

(a) Sketch the direction field for the given DE for  $-2 \leq x \leq 3$  and  $-2 \leq y \leq 2$  and, on the same grid, sketch the solution curve to the given IVP.

(b) Using a calculator, apply Euler's method with step size  $\Delta x = 0.5$  to approximate the value of  $f(3)$  where  $y = f(x)$  is the solution to the given IVP.

**2:** (a) The substitution  $u(x) = y'(x)$  and  $u'(x) = y''(x)$  transforms a second order DE of the form  $y'' = F(y', x)$  for  $y = y(x)$  to the first order DE  $u' = F(u, x)$  for  $u = u(x)$ . Use this substitution to solve the IVP  $y'' + x(y')^2 = 0$  with  $y(0) = 2$  and  $y'(0) = \frac{1}{2}$ .

(b) The substitution  $u(y(x)) = y'(x)$  and  $u'(y(x))y'(x) = y''(x)$  transforms a second order DE of the form  $y'' = F(y', y)$  for  $y = y(x)$  to the first order DE  $u' u' = F(u, y)$  for  $u = u(y)$ . Use this substitution to solve  $y'' + (y')^2 = 2e^{-y}$  with  $y(0) = 0$  and  $y'(0) = 2$ .

**3:** Given one solution  $y = y_1(x)$  to the linear homogeneous DE  $y'' + p(x)y' + q(x)y = 0$ , we can often find a second independent solution by trying  $y_2(x) = y_1(x)u(x)$  for some function  $u = u(x)$ . Use this method, known as **reduction of order**, to solve each of the following.

(a) Solve the DE  $x^3y'' + xy' - y = 0$ , given that  $y = x$  is one solution.

(b) Solve the IVP  $x^2y'' + 3xy' + y = 0$  with  $y(1) = 2$ ,  $y'(1) = 3$  given that  $y = \frac{1}{x}$  is one solution to the DE.

**4:** Given two independent solutions  $y = y_1(x)$  and  $y = y_2(x)$  to the linear homogeneous DE

$$y'' + p(x)y' + q(x)y = 0$$

we can often find a particular solution  $y = y_p(x)$  to the associated non-homogeneous DE

$$y'' + p(x)y' + q(x)y = r(x)$$

by trying  $y_p(x) = y_1(x)u_1(x) + y_2(x)u_2(x)$  for some functions  $u_1(x)$  and  $u_2(x)$  satisfying the condition  $y_1u_1' + y_2u_2' = 0$  (1). Putting  $y = y_1u_1 + y_2u_2$  into the non-homogeneous DE, and using condition (1) along with the fact that  $y_1$  and  $y_2$  are solutions to the homogeneous DE, gives  $y_1'u_1' + y_2'u_2' = r$  (2). Solving the two equations (1) and (2) allows us to find the unknown functions  $u_1$  and  $u_2$ , and hence the particular solution  $y_p = y_1u_1 + y_2u_2$ . Use this method, known as **variation of parameters**, to solve each of the following.

(a) Solve the DE  $x^2y'' - x(x+2)y' + (x+2)y = 2x^3$  given that  $y = x$  and  $y = xe^x$  are solutions to the associated homogeneous DE.

(b) Solve the DE  $xy'' - (1+x)y' + y = x^2e^{2x}$  given that  $y = 1+x$  and  $y = e^x$  are solutions to the associated homogeneous DE.