SYDE Advanced Math 2, Assignment 10 Due Fri July 26, 11:00 pm

**1:** (a) Use the method of separation of variables to find a solution $u = u(x, y)$ to the PDE $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x+y)u$ with $u(1, 0) = 4$ and $u(0, 1) = 1$.

(b) Solve the PDE given by $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = y$ for $u = u(x, y)$ with $u(x, y) = 1$ on the line $x+y = 1$ by making a change of variables, letting $r = x$ and $s = y - 2x$ (use the Chain Rule to write $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$ in terms of $\frac{\partial u}{\partial r}$, $\frac{\partial u}{\partial s}$).

**2:** (a) Use separation of variables and Fourier series to solve the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{1}{4}\frac{\partial^2 u}{\partial x^2}$ for $u = u(x, t)$ with $0 \le x \le 2$ and $t \in \mathbb{R}$ satisfying the fixed ends condition $u(0, t) = u(2, t) = 0$ for all $t \in \mathbb{R}$ and the initial conditions $u(x, 0) = (\sin \pi x)(1 + \cos \pi x)$ and $\frac{\partial u}{\partial t}(x, 0) = 0$ for all $0 \le x \le 2$.

(b) Find a constant $c$ and function $g(x)$ such that $u(x, t) = g(x + ct) + g(x - ct)$ for all $x, t$ (and show that this is the case).

(c) By plotting points, accurately sketch the graphs $u = u(x, t)$ (in the $xu$-plane) for $t = 0, \frac{1}{2}, 1, \frac{3}{2}, 2$.

**3:** (a) Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ for $u = u(x, t)$ with $0 \le x \le \pi$ and $t \ge 0$ satisfying the insulated ends condition $\frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0$ for all $t \ge 0$ and the initial condition $u(x, 0) = x^2$ for all $0 \le x \le \pi$.

(b) Give a fairly accurately sketch of the graphs of $u = u(x, t)$ (in the $xu$-plane) for $t = 0, \frac{1}{2}, 1, 10$.

**4:** (a) Define $f : \mathbb{R} \to \mathbb{R}$ by $f(z) = z^2$ and let $v, w : \mathbb{R}^2 \to \mathbb{R}$ be the real and imaginary parts of $f$ so that $f(x + iy) = v(x, y) + i\, w(x, y)$. Show that $v$ and $w$ both satisfy Laplace's equation.

(b) Solve Laplace's Equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $u = u(x, y)$ on the square $0 \le x \le 1$, $0 \le y \le 1$ satisfying the boundary conditions $u(x, 0) = x^2$, $u(x, 1) = x - 1$, $u(0, y) = -y^2$ and $u(1, y) = 1 - y^2$. Hint: use $v(x, y)$ from Part (a) and notice that $u(x, 1) \ne v(x, 1)$.