PMATH 450/650 Exercises for Chapter 3

1: (a) Let $f : [a, b] \to \mathbf{R}$ be continuous. Show that $||f||_\infty = \max_{a \leq x \leq b} |f(x)|$.

(b) Let $A \subseteq \mathbf{R}$ be measurable. Suppose that $f_n \to f$ in $L_\infty(A)$. Show that there exists $B \subseteq A$ with $\lambda(B) = 0$ such that $f_n \to f$ uniformly in $A \setminus B$.

2: (a) Show that if $1 \leq p < q \leq \infty$ then $\ell_p \subsetneq \ell_q$.

(b) Show that ℓ_p is separable for $1 \leq p < \infty$ but that ℓ_∞ is not.

3: (a) Show that $L_\infty(0, 1) \neq \bigcap_{1 < p < \infty} L_p(0, 1)$ and that $L_1(0, 1) \neq \bigcup_{1 < p < \infty} L_p(0, 1)$.

(b) Let $A \subseteq \mathbf{R}$ be measurable with $\lambda(A) < \infty$ and let $f \in L_\infty(A)$. Show that $\lim_{p \to \infty} ||f||_p = ||f||_\infty$.