

## PMATH 450/650 Exercises for Chapter 1

**1:** (a) Show that for all sets  $A \subseteq \mathbf{R}$  we have  $c^*(\overline{A}) = c^*(A)$ .  
(b) Show that for every compact set  $A \subseteq \mathbf{R}$  we have  $c^*(A) = \lambda^*(A)$ .

**2:** (a) Let  $A \subseteq \mathbf{R}$  with  $\lambda^*(A) > 0$ . Show that there is a bounded open interval  $I$  such that  $\lambda^*(A \cap I) > \frac{1}{2}|I|$ .  
(b) Let  $A \subseteq \mathbf{R}$  be bounded. Show that there is a set  $B \subseteq A$  with  $\lambda^*(B) = \frac{1}{2}\lambda^*(A)$ .

**3:** (a) Let  $A \subseteq \mathbf{R}$  be measurable with  $\lambda(A) > 0$ . Show that there exists a nonmeasurable set  $B \subseteq A$ .  
(b) Show that there exist disjoint sets  $A, B \subseteq \mathbf{R}$  such that  $\lambda^*(A \cup B) \neq \lambda^*(A) + \lambda^*(B)$ .

**4:** (a) Show that  $\mathcal{F} \subseteq \mathcal{G}_\delta$  (or, equivalently, that  $\mathcal{G} \subseteq \mathcal{F}_\sigma$ ).  
(b) Show that  $\mathcal{F}_\sigma \neq \mathcal{G}_\delta$ .  
(c) Show that  $\mathcal{G}_\delta \neq \mathcal{G}_{\delta\sigma}$  (or, equivalently, that  $\mathcal{F}_\sigma \neq \mathcal{F}_{\sigma\delta}$ ).