MATH 245 Linear Algebra 2, Exercises for Chapter 9

1: (a) Let $A = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 4 & 1 \\ 5 & 1 & 3 \end{pmatrix} \in M_3(\mathbb{Z}_7)$. Find an invertible matrix $P \in M_3(\mathbb{Z}_7)$ such that $P^T A\, P$ is diagonal.

(b) Show that in $M_3(\mathbb{Z}_7)$ we have $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cong \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ but $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ncong \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

2: (a) Let $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \\ 3 & 2 & 4 \end{pmatrix} \in M_3(\mathbb{Z}_5)$. Find a matrix $P \in M_3(\mathbb{Z}_5)$ such that $P^T AP = I$.

(b) Let $A \in M_n(\mathbb{Z}_3)$ with $A^T = A$ and $\det A = 1$. Show that there exists $P \in M_n(\mathbb{Z}_3)$ such that $P^T AP = I$.