MATH 245 Linear Algebra 2, Exercises for Chapter 4

1: (a) Let $u=(1,0,1,2)^T$, $v=(2,1,1,3)^T$ and $w=(1,2,0,1)^T$. Find $X(u,v,w)$.

(b) Let $u_1,u_2,\cdots,u_{n-2}\in\mathbb{R}^n$ with $\{u_1,u_2,\cdots,u_{n-2}\}$ linearly independent, let $A=(u_1,u_2,\cdots,u_{n-2})$ and let $U=\text{Col(A)}$. Show that for $x\in\mathbb{R}^n$ we have $\text{Proj}_{U^\perp}(x)=\dfrac{-1}{\det(A^TA)}\,X\big(u_1,\cdots,u_{n-2},X(u_1,\cdots,u_{n-2},x)\big)$.

2: (a) Find $X(u_1,u_2,\cdots,u_{n-1})$, where $u_k=e_k-k\,e_n\in\mathbb{R}^n$, for $k=1,2,\cdots,n-1$.

(b) Let U and V be hyperspaces in $\mathbb{R}^n$ with $U\neq V$. Let $W=U\cap V$ and note that W is $(n-2)$-dimensional and the spaces $U\cap W^\perp$ and $V\cap W^\perp$ are both 1-dimensional. Let $\{w_1,\cdots,w_{n-2}\}$ be a basis for W, let $\{u\}$ be a basis for $U\cap W^\perp$, and let $\{v\}$ be a basis for $V\cap W^\perp$, and note that $\{w_1,\cdots,w_{n-2},u\}$ is a basis for U and $\{w_1,\cdots,w_{n-2},v\}$ is a basis for V. Let $x=X(w_1,\cdots,w_{n-2},u)$ and $y=X(w_1,\cdots,w_{n-2},v)$, and note that $\{x\}$ and $\{y\}$ are bases for $U^\perp$ and $V^\perp$. Let $A=(w_1,\cdots,w_{n-2})\in M_{n\times(n-2)}(\mathbb{R})$. Use Theorem 4.9 to show that $x\bullet y=(u\bullet v)\det(A^TA)$, $|x|^2=|u|^2\det(A^TA)$ and $|y|^2=|y|^2\det(A^TA)$ and hence provide an alternate proof that $\theta(U^\perp,V^\perp)=\theta(U,V)$.