MATH 245 Linear Algebra 2, Exercises for Chapter 3

1: Find the least-squares best fit quadratic $f \in P_2(\mathbb{R})$ for the following data points.

x_i	-1	0	1	2	3
y_i	0	2	3	2	-2

2: (a) In $\mathbb{R}^4$, find the angle between $\langle e_1, e_2, e_3, e_4\rangle$ and $\langle a_1, a_2, a_3, a_4\rangle$, where $a_k = \sum_{i=1}^{k} e_i$.

(b) In $\mathbb{R}^n$, find the distance h_k from $\langle e_1, e_2, \cdots, e_k\rangle$ to $\langle e_{k+1}, e_{k+2}, \cdots, e_n\rangle$.

3: Let U and V be subspaces of $\mathbb{R}^n$.

(a) Show that $(U \cap V)^\perp = U^\perp + V^\perp$.

(b) Show that $\theta(U,V) = 0 \iff \theta(U^\perp, V^\perp) = 0$.

(c) Show that $\theta(U,V) = \frac{\pi}{2} \iff \theta(U^\perp, V^\perp) = \frac{\pi}{2}$.

4: Let U and V be subspaces of $\mathbb{R}^n$ with $0 < \theta(U,V) < \frac{\pi}{2}$. Let $W = U \cap V$ and $Z = U^\perp \cap V^\perp$ so that we have

$$\theta\left(U,V\right) = \min\left\{\theta(u,v)\Big|0 \neq u \in U \cap W^\perp\,,\, 0 \neq v \in V \cap W^\perp\right\}\,,\text{ and}$$

$$\theta\left(U^\perp,V^\perp\right) = \min\left\{\theta(x,y)\Big|0 \neq x \in U^\perp \cap Z^\perp\,,\, 0 \neq y \in V^\perp \cap Z^\perp\right\}.$$

Choose $u \in U \cap W^\perp$ and $v \in V \cap W^\perp$ with $|u| = |v| = 1$ such that $\theta(u,v) = \theta(U,V)$.

(a) Show that $\text{Proj}_V(u) = \text{Proj}_v(u) = (u \cdot v)\, v$ and $\text{Proj}_U(v) = \text{Proj}_u(v) = (u \cdot v)\, u$.

(b) Let $x = (u \cdot v)u - v$ and $y = u - (u \cdot v)v$. Show that $0 \neq x \in U^\perp \cap Z^\perp$, $0 \neq y \in V^\perp \cap Z^\perp$ and $\theta(x,y) = \theta(u,v)$.

(c) Show that $\theta(U^\perp, V^\perp) = \theta(U,V)$.