MATH 245 Linear Algebra 2, Exercises for Chapter 2

1: (a) Let $u_1 = (1,0,2,1)^T$, $u_2 = (1,1,3,2)^T$, $u_3 = (1,-1,2,0)^T$ and $x = (3,2,-1,2)^T$. Let $\mathcal{A} = \{u_1, u_2, u_3\}$ and let $U = \text{Span}\,\mathcal{A}$. Find $\text{Proj}_U(x)$.

(b) Let $A = \begin{pmatrix} 1 & 2 & 1 & 3 & 4 \\ 2 & 3 & 1 & 5 & 6 \\ 1 & 1 & 0 & 1 & 3 \\ -1 & 1 & 2 & 2 & 0 \end{pmatrix} \in M_{4\times 5}(\mathbb{R})$ and $x = \begin{pmatrix} 2 \\ 1 \\ 4 \\ 1 \\ 2 \end{pmatrix} \in \mathbb{R}^5$. Find $\text{Proj}_{\text{Null}(A)}(x)$.

2: (a) Let $A \in M_{k\times n}(\mathbb{R})$ with $\text{rank}(A) = k$, and let $b \in \mathbb{R}^n$. Find a formula, in terms of A and b, for the point $x \in \mathbb{R}^n$ with $Ax = Ab$ which is nearest to the origin.

(b) Find the point $x \in \mathbb{R}^4$, of minimum possible norm, such that $Ax = b$, where

$$A = \begin{pmatrix} 1 & 2 & 1 & -1 \\ 2 & 3 & 1 & -1 \\ 1 & 0 & -1 & 1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}.$$

3: (a) Let $0 \neq u, v, w \in \mathbb{R}^n$. Suppose that $w = su + tv$ with $s, t \geq 0$. Show that $\theta(u,v) = \theta(u,w) + \theta(w,v)$.

(b) Let $[a,b,c]$ be a triangle in $\mathbb{R}^n$. Let $\alpha = \angle bac$, $\beta = \angle cba$ and $\gamma = \angle acb$. Show that $\alpha + \beta + \gamma = \pi$.