MATH 245 Linear Algebra 2, Exercises for Chapter 12

1: Find the number of similarity classes of 8×8 complex matrices whose distinct eigenvalues are 1 and 2.

2: Let $A = \begin{pmatrix} 2 & -7 & 1 & 7 \\ 3 & -8 & 2 & 7 \\ 0 & 1 & -2 & -1 \\ 3 & -8 & 3 & 7 \end{pmatrix}$. Find an invertible matrix P such that $P^{-1}AP$ is in Jordan form.

3: Let $A = \begin{pmatrix} 1 & 1 & 0 & & \\ 0 & 1 & 1 & & \\ 0 & 0 & 1 & & \\ & & & -1 & 1 \\ & & & 0 & -1 \end{pmatrix}$.

(a) Find A^n, where n is a positive integer.

(b) Find $e^A = I + A + \frac{1}{2!}A^2 + \frac{1}{3!}A^3 + \cdots$.

4: Let $A \in M_{n\times n}(\mathbb{C})$.

(a) Show that A is similar to A^T.

(b) Show that if A is invertible then there is a matrix $B \in M_{n\times n}(\mathbb{C})$ such that $A = B^2$.

(c) Show that for $n \geq 2$, there is no matrix $B \in M_{n\times n}(\mathbb{C})$ such that $J_0^n = B^2$.