MATH 245 Linear Algebra 2, Exercises for Chapter 10

1: (a) Find the vertices and the asymptotes of the hyperbola $11x^2 + 24xy + 4y^2 = 500$.

(b) Find the volume of the ellipsoid $2x^2 + 3y^2 + 3z^2 + 2xy + 2xz = 9$.

2: (a) Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$. Find $\max\limits_{|x|=1} |Ax|$ and $\min\limits_{|x|=1} |Ax|$. Find $u \in \mathbb{R}^3$ with $|u| = 1$ so that $|Au| = \min\limits_{|x|=1} |Ax|$.

(b) Define $L : \mathbb{R}^\infty \to \mathbb{R}^\infty$ by $L(x_1, x_2, x_3, x_4, \cdots) = \left(\sum\limits_{i=1}^{\infty} x_i \,,\, x_2, x_3, x_4, \cdots \right)$. Find $\sup\limits_{|x|=1} |Lx|$ and $\inf\limits_{|x|=1} |Lx|$.

3: Let U and V be non-trivial subspaces of $\mathbb{R}^n$ with $U \cap V = \{0\}$. Recall that

$$\theta(U, V) = \min \left\{ \theta(u, v) \,\middle|\, 0 \neq u \in U, 0 \neq v \in V \right\}.$$

(a) Show that $\theta(U, V) = \cos^{-1}(\sigma)$ where σ is the largest singular value of the linear map $P : U \to V$ given by $P(x) = \text{Proj}_V(x)$.

(b) Let $u_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$, $u_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \\ 2 \end{pmatrix}$, $v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}$. Let $U = \text{Span}\{u_1, u_2\}$ and $V = \text{Span}\{v_1, v_2\}$.
Find $\theta(U, V)$.