MATH 239 Intro to Combinatorics, Assignment 5 Due Mon Apr 2

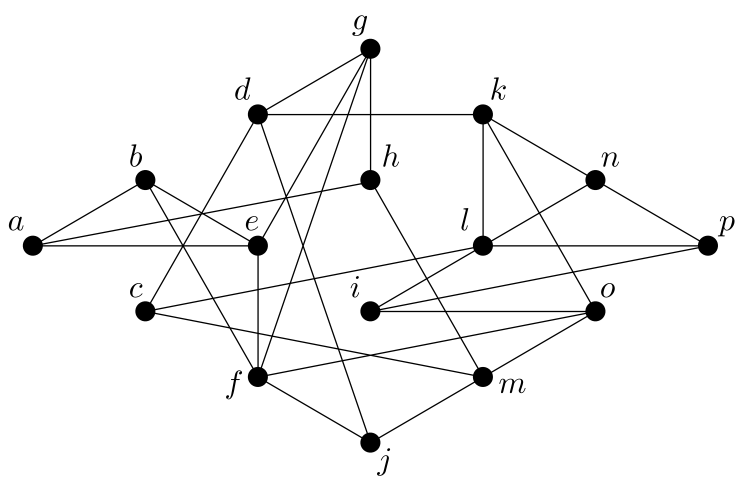
1: (a) Up to isomorphism, there are 25 trees with at most 7 vertices. Without proof, draw a picture of one tree from each of these 25 isomorphism classes.

(b) Without proof, find the number of connected graphs G with 6 vertices and 6 edges, up to isomorphism. Draw a picture of one graph from each isomorphism class.

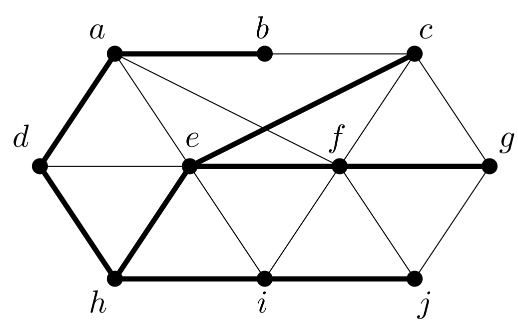
2: (a) Show that in any tree whose vertices all have odd degree, the number of vertices of degree 1 is greater than the number of vertices whose degree is not equal to 1.

(b) Show that given any positive integers $d_1, d_2, \cdots, d_p$ with $d_1 + d_2 + \cdots + d_p = 2p - 2$, there exists a tree with p vertices $v_1, v_2, \cdots, v_p$ with $\deg(v_i) = d_i$ for all i.

3: (a) Find a breadth-first search spanning tree, rooted at vertex a, in the graph shown below and use your tree to find a shortest path from a to p.



(b) The picture below shows a spanning tree T (with edges in bold face) in a graph G. Determine whether it is possible to obtain T using a breadth-first search in G and, if so, determine the order in which the vertices were added to the tree.



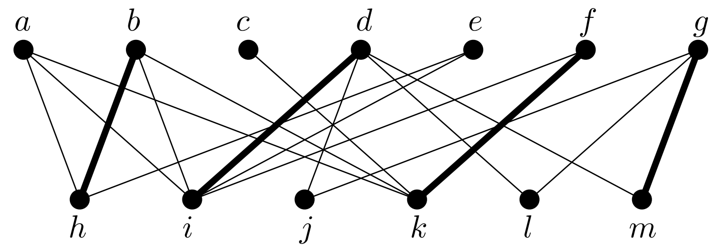
4: (a) Find the number of perfect matchings in the 3-cube Q_3.

(b) Find the number of perfect matchings in the complete bipartite graph $K_{n,n}$.

(c) Find the number of perfect matchings in the complete graph K_{2n}.

5: For each of the following two graphs, use the bipartite matching algorithm, starting with the matching given by the bold edges, to find a maximum matching and a minimum cover.

(a)



(b)