MATH 239 Intro to Combinatorics, Assignment 3 Due Thurs Feb 16

1: Given $n, k \in \mathbf{N} = \{0, 1, 2, \cdots\}$, find the number of sequences $(a_1, a_2, \cdots, a_{2k})$ with each $a_i \in \mathbf{N}$ such that $a_1 + a_2 + \cdots + a_k + 2a_{k+1} + 2a_{k+2} + \cdots + 2a_{2k} = n$. In particular, what is the number of such sequences when $k = 5$ and $n = 7$?

2: For a positive integer n, let c_n be the number of compositions of n into an even number of parts each of which is odd. Find the generating function $\sum c_n x^n$ expressed as a rational function, obtain a recursion formula for c_n, and use the recursion formula to find c_{10}.

3: Given positive integers n and k, find the number of integer sequences $(a_1, a_2, \cdots, a_k)$ with $1 \leq a_1 < a_2 < \cdots < a_k \leq n$ such that $a_i = i \pmod 3$ for all i. Express your answer in simplified form using the floor function.

4: For each $n \in \mathbf{N}$, let c_n be the number of binary strings of length n which do not contain either 0000 or 1111 as substrings. Find the generating function $\sum c_n x^n$ expressed as a rational function, obtain a recursion formula for c_n, and find c_6.

5: Find the generating function with respect to length, expressed as a rational function, for the set of binary strings in which no 0-block is followed by a 1-block of greater length. In particular, find the number of such sequences of length 8.