MATH 239 Intro to Combinatorics, Assignment 1 Due Mon Jan 23

1: Given a positive integer n, find the number of sequences $(a_1, a_2, \cdots, a_n)$, with $a_i \in \{1, 2, 3\}$ for all i, which do not contain 11, 22 or 33 as substrings.

2: Given a positive integer n, find the number of binary sequences of length n which do not contain either 000 or 111 as a subsequence.

3: Given positive integers n and k, determine the number of sequences $(a_1, a_2, \cdots, a_k)$, with $-1 \leq a_i \in \mathbf{Z}$ for all i, such that $a_1 + a_2 + \cdots + a_k = n$.

4: Given a positive integer $n \geq 2$, show that the total number of sequences $(a_1, a_2, \cdots, a_k)$ with $1 \leq k$, $2 \leq a_i \in \mathbf{Z}$ for all i, and $a_1 + a_2 + \cdots + a_k = n$ is equal to F_{n-1}, the $(n-1)^{\text{st}}$ Fibonacci number.

5: Let c_n be the number of sequences $(a_1, a_2, \cdots, a_n)$, with each $a_i \in \{1, 2\}$, such that we have $a_1 + a_2 + \cdots + a_k = n$ for some $k \leq n$.

(a) Show that $c_n = c_{n-1} + 2c_{n-2}$.

(b) Find a formula for c_n in terms of n.