Name (print): _____

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MATH 218, Differential Equations for Engineers

Midterm Test, Fall Term, 2008

University of Waterloo

Instructor: Stephen New

Date: October 30, 2008

Time: 8:30-10:20 am

Question	Mark
1	/5
2	/5
3	/5
4	/5
5	/5
6	/5
7	/5
8	/5
Total	/40

Instructions:

1. Place your name, signature and ID number in the spaces provided at the top of this page.

2. This test contains 9 pages, including this one.

3. No calculators are allowed.

4. Answer all 8 questions.

5. Provide full explanations with all your solutions.

[5] **1:** (a) Solve the IVP $x\,y' + 2y = 6$ with $y(1) = 2$.

(b) Solve the IVP $y' = \dfrac{x}{y} + \dfrac{y}{x}$ with $y(1) = 2$.

[5] **2:** In a chemical reaction, 1 g of substance A reacts with 1 g of substance B to produce 2 g of substance C. Suppose that 2 g of substance A and 3 g of substance B are combined at time $t = 0$ seconds. Let $a(t)$, $b(t)$ and $c(t)$ be the amounts, in grams, of the three substances at time t, and suppose that $c(t)$ satisfies the DE $c'(t) = 4a(t)b(t)$. Find a formula for $c(t)$.

[5] **3:** Solve the IVP $x^2y'' = \left(y'\right)^2$ with $y(1) = 3$ and $y'(1) = 2$.

[5] **4:** Find the general solution to the DE $x^2y'' + 3xy' + y = 0$ for $x > 0$, given that $y_1(x) = \frac{1}{x}$ is one solution to the DE.

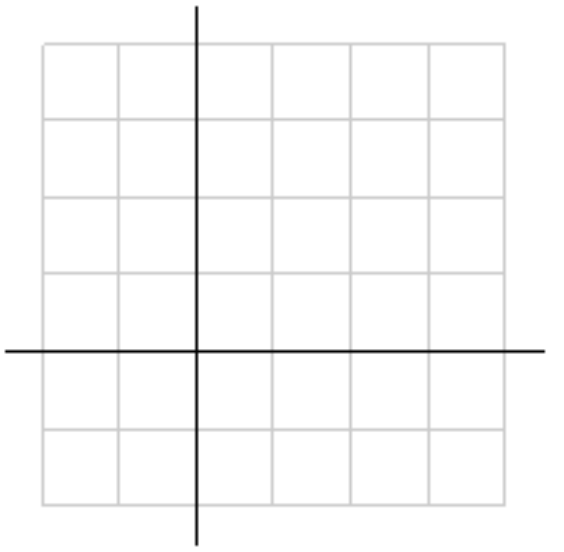
[5] **5:** Solve the DE $4y'' - 4y' + y = x\,e^x$.

[5] **6:** An object of mass $m = 1$ kg is attached to a spring of spring-constant $k = 5$ N/m in a liquid where the damping-constant is $c = 2$ kg/s. A constant driving force of $F = 5$ N is applied to the object. Let $x(t)$ be the displacement of the object from the equilibrium position. Given that $x(0) = 0$ and $x'(0) = 0$, find the maximum value of x.

[5] **7:** Use the Power Series Method to find the solution of the IVP $y'' - 2x\,y' - 2y = 0$ with $y(0) = 1$, $y'(0) = 0$. First find a power series solution, then convert the power series to closed form.

[5] **8:** Consider the DE $y' = \dfrac{y-x}{y-1}$.

(a) Sketch the direction field for the given DE along with the solution curves through each of the points $(0,-1)$, $(0,2)$ and $(4,2)$.



(b) Use Euler's Method with step size $\Delta x = 1$ to approximate the value of $f(3)$ where $y = f(x)$ is the solution to the given DE with $y(0) = 2$.