MATH 218 Differential Equations, Assignment 9 Due Tues Nov 18

**1:** Use Laplace transforms to solve the following IVPs.

(a) $y' - 2y = e^{2t}$ with $y(0) = 3$.

(b) $y'' - 6y' + 13y = 0$ with $y(0) = 2$, $y'(0) = 0$.

**2:** Use Laplace transforms to solve the following IVPs.

(a) $y'' + 2y' + y = e^{-t}$ with $y(0) = 1$, $y'(0) = 2$.

(b) $2y'' - 3y' - 2y = 5e^{2t}$ with $y(0) = 1$, $y'(0) = 2$.

**3:** Use Laplace transforms to solve the following IVPs.

(a) $y'' + 3y' + 2y = g(x)$ with $y(0) = 0$ and $y'(0) = 0$, where $g(x) = \begin{cases} 2 & \text{, if } 0 \le t \le 2, \\ 6 - 2t & \text{, if } 2 \le t \le 3, \\ 0 & \text{, if } 3 \le t\,. \end{cases}$

(b) $2y''' - 3y'' + 0y' + y = 0$ with $y(0) = 1$, $y'(0) = -3$ and $y''(0) = 2$.

**4:** A tank initially contains 20 L of pure water. Brine pours in at the rate of 4 L/min. The concentration (in g/L) of this brine at time $t$ minutes is given by

$$c(t) = \begin{cases} t & \text{, if } 0 \le t \le 5, \\ 10 - t & \text{, if } 5 \le t \le 10, \\ 0 & \text{, if } 10 \le t\,. \end{cases}$$

The tank is kept well mixed, and brine drains from the tank at 4 L/min. Use Laplace transforms to find the amount of salt $y(t)$ in the tank at time $t$, and in particular, find the time at which the amount of salt reaches its maximum.

**5:** An object of mass $m = 1$ kg is attached to a spring of spring-constant $k = 5$ N/m in a liquid where the damping constant is $c = 2$ kg/s. The object is initially at rest at the equilibrium position, then it is struck repeatedly with a hammer, first at time $t = 0$, and then again every $\pi$ seconds. Each strike of the hammer imparts an impulse which increases the object's momentum by 1 kg m/s$^2$. Determine the motion of the object, and in particular, find the asymptotic (or eventual) velocity of the object just after each strike.

**6:** (Not to hand in)

(a) Find the inverse Laplace transform of each of the following functions $F(s)$.

(i) $F(s) = \dfrac{8}{(s^2 + 2s + 2)^3}$. (ii) $F(s) = \dfrac{8s}{(s^2 + 2s + 2)^3}$.

(b) Find a formula for the solution to the IVP $y'' + 2y' + 5y = g(x)$ with $y(0) = 2$, $y'(0) = 4$.