MATH 218 Differential Equations, Assignment 8 Due Tues Nov 11

**1:** Find the Laplace transform of each of the following functions $f(t)$.

(a) $f(t) = t^3 e^{-t/2}$.

(b) $(t + e^t)^3$.

(c) $f(t) = a\sinh(\omega t) + b\cosh(\omega t)$.

**2:** Find the Laplace transform of each of the following functions $f(t)$.

(a) $f(t) = \begin{cases} t & , \text{if } 0 \le t < 1, \\ 3 - 2t & , \text{if } 1 \le t < 2, \\ t - 3 & , \text{if } 2 \le t < 3, \\ 0 & , \text{if } 3 \le t\,. \end{cases}$

(b) $f(t) = 2e^t \cos\left(2t - \frac{\pi}{3}\right)$.

(c) $f(t) = t\, e^{-t} \sin 3t$.

**3:** Find the inverse Laplace transform of the following functions $F(s)$.

(a) $F(s) = \dfrac{s-3}{s^3 + s^2 + s + 1}$.

(b) $F(s) = \dfrac{5s+7}{s^3 + 4s^2 + s - 6}$.

(c) $F(s) = \dfrac{s-2}{s^4 + 4s^3 + 8s^2}$.

**4:** Find the inverse Laplace transform of the following functions $F(s)$.

(a) $F(s) = \dfrac{(s+1)e^{-s}}{s(s+2)}$.

(b) $F(s) = \dfrac{4 + se^{-2s}}{s^2 - 4}$.

(c) $F(s) = \dfrac{s^2 - 1}{(s^2+1)^2}$.

**5:** (a) Show that if $f(t)$ is periodic with period $T$ (this means that $f(t+T) = f(t)$ for all $t$) then

$$\mathcal{L}\big[f(t)\big](s) = \frac{1}{1 - e^{-Ts}} \int_0^T f(t) e^{-st}\, dt\,.$$

(b) Find the Laplace transform of the sawtooth function $f(t)$ which is given by $f(t) = t - n$ for $n \le t < n + 1$ for each integer $n \ge 0$.