MATH 218 Differential Equations, Assignment 6 Due Tues Oct 21

1: An object of mass $m = 2$ kg is attached to a spring with spring-constant $k = 18$ N/m. Let $x(t)$ be the object's displacement, in meters, from the equilibrium position at time t seconds. Suppose that $x(0) = 2$ and $x'(0) = 6$. Ignoring air resistance, find the smallest positive value of t such that $x(t) = 0$.

2: An object of mass $m = 2$ kg is attached to a spring with spring-constant $k = 8$ N/m in a liquid where the damping-constant is $c = 10$ kg/s. Let $x(t)$ be the displacement, in meters, from the equilibrium position at time t seconds.

(a) Determine whether the system is overdamped, underdamped, or critically damped.

(b) Given that $x(0) = 1$ and $x'(0) = 5$, find the time t at which $x(t)$ reaches its maximum.

3: An LRC series circuit consists of an inductor of inductance $L = 1$ henries, a resistor of resistance $R = 2$ ohms, and a capacitor of capacitance $C = \frac{1}{5}$ farads, and there is an impressed voltage of $E = \sin 2t$ volts. Let $q = q(t)$ coulombs be the charge on the capacitor at time t seconds.

(a) Find the asymptotic (that is limiting or eventual) amplitude and phase of $q(t)$.

(b) Given that $q(0) = 0$ and $q'(0) = 0$, find a formula for $q(t)$.

4: On a planet where the surface gravitational constant happens to be $g = 10$ m/s^2, an object of mass $m = 1$ kg is thrown upwards from the ground at 20 m/s. Let $x(t)$ be its height in meters at time t seconds. Suppose the force at time t due to air resistance is $-\frac{1}{10}\,x'(t)$ newtons.

(a) Find the time t at which the object reaches its maximum height.

(b) Find $x(t)$ and (with the help of a calculator) determine whether the object takes longer on the way up to its maximum height or on the way back down to the ground.

5: An object of mass m falls towards the Earth. The force due to gravity is $F = -\dfrac{GMm}{x^2}$ where x is the distance from the center of the Earth to the object, G is the gravitational constant and M is the mass of the Earth.

(a) Given that $x(0) = x_0$ and $x'(0) = 0$, find a formula for the velocity x' in terms of x.

(b) Find a formula for time t in terms of x, and then find the time at which $x = \frac{1}{2}\,x_0$.