MATH 218 Differential Equations, Assignment 12 Not to hand in

1: Two tanks A and B are connected by pipes. Tank A initially contains 8 L of brine with a salt concentration of 4 g/L. Tank B initially contains 6 g of pure water. Brine with a salt concentration 3 g/L enters tank A at the rate 5 L/min. Brine flows from tank A to tank B at the rate 3 L/min through one pipe, and brine flows back from tank B to tank A at the rate of 1 L/min through another pipe. Also, brine is drained from tank A at 1 L/min and from tank B at 2 L/min. Find the amount of salt in each tank as a function of time.

2: A tank contains four chemicals, A, B, C and D. Three chemical reactions take place, namely $A + B \to C$, $C \to A + B$ and $A + C \to D$. The molarities a, b, c and d of these chemicals satisfy the following system of DEs

$$\begin{aligned} a' &= -kab - lac + mc \\ b' &= -kab + mc \\ c' &= kab - lac - mc \\ d' &= lac \end{aligned}$$

where $k = \frac{1}{200}$, $l = \frac{1}{300}$ and $m = \frac{1}{6}$. The initial molarities are $a(0) = 100$, $b(0) = 3$, $c(0) = 1$ and $d(0) = 0$. This system is not linear, so it is difficult to solve, but since the amount $a(t)$ is relatively large, make the simplifying assumption that $a(t) = 100$ is constant, then solve the resulting system of DEs for $b(t)$, $c(t)$ and $d(t)$.

3: (a) Solve the system $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 6t+1 \\ e^t \end{pmatrix}$.

(b) Solve the system $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 5 \\ 2e^t \end{pmatrix}$.

4: Find the solution to the system $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 2 & -2 & 1 \\ 1 & -1 & 1 \\ 2 & -4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ with $\begin{pmatrix} x(0) \\ y(0) \\ z(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

5: Solve the system $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 1 & -2 \\ -2 & -2 & 2 \\ 3 & 2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} e^{-t}$.