MATH 218 Differential Equations, Assignment 10 Due Tues Nov 25

1: Consider the system $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} xy \\ x+y \end{pmatrix}$.

(a) Sketch the isoclines $m = 0, \pm\frac{1}{2}, \pm 1, \pm 2$ and sketch the slope field for this system.

(b) Use Euler's method with step size $\Delta t = \frac{1}{2}$ to approximate the point $\big(x(2), y(2)\big)$, where $\big(x(t), y(t)\big)$ is the solution to the above system with $\big(x(0), y(0)\big) = (-1, 1)$.

2: Consider the system $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{1}{y} \\ \frac{2}{x} \end{pmatrix}$.

(a) Sketch the phase diagram for this system.

(b) Solve the system by eliminating y and y' from x'' to get a second order DE for $x = x(t)$.

(c) In particular, find the solution to the system with $\big(x(0), y(0)\big) = (2, 1)$.

3: Consider the system $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} xy \\ \frac{1}{2}(y^2 - x^2) \end{pmatrix}$.

(a) Sketch the phase diagram for this system.

(b) Find the solution $\big(x(t), y(t)\big)$ to the above system with $\big(x(-1), y(-1)\big) = (1, 1)$ by first finding the function $f(x)$ such that the solution curve lies on the graph $y = f(x)$.

4: Use the method of reduction of order to solve the system $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{t} \\ \frac{1}{t} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ given that $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{t} \\ -\frac{1}{t} \end{pmatrix}$ is one solution.

5: Use reduction of order and variation of parameters to solve $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{t^2} \\ 1 & \frac{1}{t} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 4 \\ 3\sqrt{t} \end{pmatrix}$ given that $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{t} \\ -1 \end{pmatrix}$ is one solution to the associated homogeneous system.