MATH 218 Differential Equations, Assignment 1 Due Tues Sept 16

1: Find each of the following integrals.

(a) $\displaystyle\int_0^{\pi/3} \frac{\sin^3 x}{\cos^2 x}\,dx$ (b) $\displaystyle\int_0^3 \frac{x^2\,dx}{(x+1)^{3/2}}$ (c) $\displaystyle\int_0^{\pi} (3x+1)\cos(x/2)\,dx$ (d) $\displaystyle\int_0^{\pi/2} e^{2x}\cos x\,dx$

2: Find each of the following integrals.

(a) $\displaystyle\int_0^{\sqrt{3}} \frac{x^2\,dx}{\sqrt{4-x^2}}$ (b) $\displaystyle\int_0^{\infty} \frac{dx}{(x^2+2x+4)^{3/2}}$ (c) $\displaystyle\int_1^3 \frac{x^4+3x^3+6}{x^3+4x^2+3x}\,dx$ (d) $\displaystyle\int_1^2 \frac{x^3+2}{x^5+2x^3+x}\,dx$

3: (a) Verify that $y = x\sin x$ is a solution of the DE $y\,(y''+y) = x\sin 2x$.

(b) Find all the solutions of the form $y = ax^2+bx+c$ to the DE $\big(y'(x)\big)^2+4x = 3y(x)+x^2+1$.

4: Consider the IVP $y' = x - y^2$ with $y(0) = 0$.

(a) Sketch the direction field for the given DE for $-2 \le x \le 3$ and $-2 \le y \le 2$.

(b) On the same grid, sketch the solution curve to the given IVP.

(c) Using a calculator, apply Euler's method with step size $\Delta x = 0.5$ to approximate the value of $f(3)$ where $y = f(x)$ is the solution to the given IVP.

5: Consider the IVP $y' = \sin\big(\pi(x+y)\big)$ with $y(-1) = 1$.

(a) Sketch the direction field for the given DE for $-2 \le x \le 2$ and $-2 \le y \le 2$.

(b) On the same grid, sketch the solution curves which pass through each of the points $(-1,1)$, $(0,0)$ and $(0,-1)$.

(c) Using a calculator, apply Euler's method with step size $\Delta x = 0.2$ to approximate the value of $f(0)$ where $y = f(x)$ is the solution to the given IVP.