MATH 148 Calculus 2, Exercises for Chapter 2

**1:** (a) Approximate $\displaystyle\int_1^3 \frac{dx}{x}$ using Simpson's Approximatoion $S_4$ and find a bound on the error.

(b) Find a value of $n$ such that if we approximate $\displaystyle\int_0^1 \frac{4\,dx}{1+x^2}$ by $M_n$, the error is $E_n \leq \frac{1}{300}$.

(c) Let $f$ be differentiable on $[a,b]$ with $|f'(x)| \leq M$ for all $x \in [a,b]$. Let $I = \displaystyle\int_a^b f$ and let $L_n$ be the Left Endpoint Approximation for $I$ on $n$ equal-sized subintervals. Prove that

$$|I - L_n| \leq \frac{M(b-a)^2}{2n}.$$

**2:** Find each of the following indefinite integrals (antiderivatives).

(a) $\displaystyle\int \frac{dx}{x\,(\ln x)^3}$, where $x > 1$ (b) $\displaystyle\int e^{-x}\cos(2x)\,dx$

(c) $\displaystyle\int \sin(\sqrt{x})\,dx$, where $x > 0$ (d) $\displaystyle\int \sqrt{\frac{1+x}{1-x}}\,dx$, where $0 \leq x < 1$

**3:** Find each of the following indefinite integrals (antiderivatives).

(a) $\displaystyle\int x^2\sqrt{4-x^2}\,dx$ (b) $\displaystyle\int \left(x\ln x\right)^2 dx$ (c) $\displaystyle\int \cos^4 x\,dx$ (d) $\displaystyle\int \frac{x^4+3x^3-x^2+1}{x^4+2x^3+x^2}\,dx$

**4:** Find each of the following definite integrals.

(a) $\displaystyle\int_0^4 |x^2-3x+2|\,dx$ (b) $\displaystyle\int_0^{\ln 3} \frac{e^x\,dx}{1+e^x}$ (c) $\displaystyle\int_0^{\pi/3} \frac{\sin^3 x}{\cos^2 x}\,dx$ (d) $\displaystyle\int_0^2 x\,e^{-x/2}\,dx$

**5:** Find each of the following definite integrals.

(a) $\displaystyle\int_1^e (\ln x)^2\,dx$ (b) $\displaystyle\int_1^3 \frac{\sqrt{x}}{x+1}\,dx$ (c) $\displaystyle\int_0^{\pi/2} \sqrt{1-\sin x}\,dx$ (d) $\displaystyle\int_1^{27} \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

**6:** Find each of the following definite integrals.

(a) $\displaystyle\int_0^3 \frac{x^2\,dx}{(x+1)^{3/2}}$ (b) $\displaystyle\int_0^{\pi/4} \tan^5 x\,dx$ (c) $\displaystyle\int_0^{\pi/2} e^{2x}\cos x\,dx$ (d) $\displaystyle\int_0^1 x^2\sin^{-1} x\,dx$

**7:** Find each of the following definite integrals.

(a) $\displaystyle\int_0^{\sqrt{3}} \frac{x^2\,dx}{(4-x^2)^{3/2}}$ (b) $\displaystyle\int_0^1 x^2\sqrt{2-x^2}\,dx$ (c) $\displaystyle\int_{\sqrt{3}}^{\sqrt{8}} \frac{\sqrt{1+x^2}}{x}\,dx$ (d) $\displaystyle\int_3^5 \frac{\sqrt{x^2-9}}{x^2}\,dx$

**8:** Find each of the following definite integrals, where $n$ is a positive integer.

(a) $\displaystyle\int_{-\pi/6}^{\pi/6} \sin(2x)\sin(3x)\,dx$ (b) $\displaystyle\int_0^{\pi^2} \sin^2\sqrt{x}\,dx$ (c) $\displaystyle\int_0^{\pi/2} \cos^{2n} x\,dx$ (d) $\displaystyle\int_0^{\pi} \frac{\sin(nx)}{\sin x}\,dx$

**9:** Find each of the following definite integrals.

(a) Find $\displaystyle\int_2^3 \frac{x-7}{(x-1)^2(x+2)}\,dx$. (b) Find $I = \displaystyle\int_1^2 \frac{x^5+x^4-2x^3-2x^2-5x-25}{x^2(x^2-2x+5)^2}\,dx$.

(c) $\displaystyle\int_1^3 \frac{x^4+3x^3+6}{x^3+4x^2+3x}\,dx$ (d) $\displaystyle\int_1^2 \frac{x^3+2}{x^5+2x^3+x}\,dx$

**10:** Find each of the following integrals.

(a) $\displaystyle\int_0^{\pi/2} \frac{\sin x\,dx}{\sin x+\cos x}$ (b) $\displaystyle\int_{\pi/4}^{\pi/2} \frac{x\,dx}{(\sin x+\cos x)^2}$ (c) $\displaystyle\int_{-\pi/4}^{\pi/2} \frac{dx}{(3\sin x+4\cos x)^2}$ (d) $\displaystyle\int_0^{\pi} \frac{x\sin x}{1+\sin^2 x}\,dx$

**11:** Find each of the following definite integrals.

(a) $\displaystyle\int_1^2 (1+2x^2)e^{x^2}\,dx$ (b) $\displaystyle\int_1^3 \frac{\sqrt[3]{5-x}\,dx}{\sqrt[3]{5-x}+\sqrt[3]{1+x}}$ (c) $\displaystyle\int_0^2 \sqrt{x^3+1}+\sqrt[3]{x^2+2x}\,dx$ (d) $\displaystyle\int_0^1 \frac{\ln(x+1)}{x^2+1}\,dx$

**12:** Find the following improper integrals.

(a) $\displaystyle\int_1^\infty \frac{dx}{x^3\sqrt{x^2-1}}$ (b) $\displaystyle\int_0^\infty \frac{e^x+1}{e^{2x}+1}\,dx$ (c) $\displaystyle\int_3^\infty \frac{dx}{(x^2-1)\sqrt{x}}$

**13:** Find each of the following improper integrals.

(a) $\displaystyle\int_{-\infty}^0 \big((x+1)e^x\big)^2\,dx$ (b) $\displaystyle\int_0^\infty \frac{dx}{\sqrt{e^x-1}}$ (c) $\displaystyle\int_{-\infty}^\infty \frac{x^3+4x+9}{(x^2+1)^2(x^2+4)}\,dx$

**14:** Evaluate the following improper integrals.

(a) $\displaystyle\int_0^{\pi/2} \sqrt{2\tan x}\,dx$ (b) $\displaystyle\int_2^{17/4} \sqrt{\frac{x+2}{x-2}}\,dx$ (c) $\displaystyle\int_0^{\ln 2} \sqrt{\frac{e^x+1}{e^x-1}}\,dx$

**15:** Test each of the following improper integrals for convergence.

(a) $\displaystyle\int_1^\infty \frac{dx}{e^x\ln x}$ (b) $\displaystyle\int_0^\infty 2^{1/x}-1\,dx$ (c) $\displaystyle\int_1^\infty \frac{dx}{(\ln x)^{\ln x}}$ (d) $\displaystyle\int_0^\infty \sin(x)\sin(x^2)\,dx$

**16:** (a) Let $\displaystyle f(x)=\int_0^x \frac{\sin(2x-t)}{2x-t}\,dt$. Find $f'\left(\frac{\pi}{2}\right)$.

(b) Let $\displaystyle f(x)=\int_0^{x^3} e^{t^2}\,dt$. Show that $\displaystyle 6\int_0^1 x^2 f(x)\,dx - 2\int_0^1 e^{x^2}\,dx = 1-e$.

(c) Let $f$ be continuous. Show that if $f\geq 0$ and $\displaystyle\int_1^3 f(x)\,dx=1$ then $\displaystyle\int_1^9 f(\sqrt{x})\,dx\leq 6$.

(d) Let $f$ be integrable with $\displaystyle\int_0^1 f(x)\,dx=\int_0^1 xf(x)\,dx=1$. Show that $\displaystyle\int_0^1 f(x)^2\,dx\geq 4$.