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Name (print): ____________________________

Signature: ____________________________

ID Number: ____________________________

Instructions:

1. Place your name, signature and ID number, in the spaces provided above.

2. This test contains 8 pages, including this cover page and two pages at the end for extra space, if needed.

3. No calculators or any other electronic devices are allowed.

4. Answer all 5 questions; all questions will be given equal value.

5. Provide full explanations with all your solutions. If you run out of space then continue on the last two pages.

Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

[10] **1:** (a) Find $\displaystyle\int_0^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

(b) Find $\displaystyle\int_0^2 \frac{x^2}{\sqrt{4-x^2}}\,dx$.

(c) Find $\displaystyle\int_1^\infty \frac{3x-2}{x^3+2x^2}\,dx$.

[10] **2:** (a) Find the area of the region given by $0 \leq x \leq \pi$, $0 \leq y \leq \sin^5 x$.

(b) Let R be the region given by $0 \leq x \leq \frac{\pi}{2}$, $0 \leq y \leq \cos x$. Find the volume of the solid obtained by revolving R about the y-axis.

(c) Let C be the curve given by $y = x^2$ with $0 \leq x \leq \sqrt{6}$. Find the area of the surface obtained by revolving C about the y-axis.

[10] **3:** (a) Solve the initial value problem given by $y' = 3\sqrt{xy}$ with $y(1) = 4$.

(b) Solve the initial value problem given by $y' = x + y + 1$ with $y(0) = 1$.

(c) A tank initially contains 2 L of pure water. Brine (salty water), with a salt concentration of 3 gm/L, enters the tank at a rate of $r(t) = \frac{1}{t+1}$ L/min, where t is the time in minutes. The brine in the tank is kept well mixed, and drains from the tank at the same rate $r(t)$. Determine when the concentration of brine in the tank is 2 gm/L.

[10] **4:** (a) Determine, with proof, whether $\sum_{n\geq 2} \dfrac{1}{(\ln n)^2}$ converges.

(b) Prove that if $\sum_{n\geq 1} |a_n|$ converges then $\sum_{n\geq 1} a_n$ converges.

(c) Let $a_n > 0$ for all $n \in \mathbb{Z}^+$ and suppose that $\lim_{n\to\infty} \dfrac{a_{n+1}}{a_n} = r$ with $0 \leq r < 1$. Prove that $\sum_{n\geq 1} a_n$ converges.

[10] **5:** (a) Find the Taylor polynomial of degree 3 centred at 0 for $f(x) = e^x\sqrt{1+2x}$.

(b) Approximate the value of $\ln\frac{3}{4}$ so that the absolute error is $E \leq \frac{1}{100}$.

(c) Evaluate $\sum_{n=1}^{\infty} \frac{n^2\, 2^n}{n!}$.

Use this page to continue solutions if you require additional space.
If you do, then clearly indicate which questions you are continuing.

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