Name (print): ______________________________

Signature: ______________________________

ID Number: ______________________________

Section (circle): 1 2 3 4 5 6 7 9 10 11

MATH 137, Calculus 1 for Honours Mathematics

Faculty of Mathematics, University of Waterloo
Final Examination, Fall Term 2012
Date: Monday, December 10th
Time: 9:00 - 11:30 am

Section	Time	Instructor
1	8:30-9:20	F. Zorzitto
2	9:30-10:20	M. Eden
3	10:30-11:20	J. Lawrence
4	11:30-12:20	M. Eden
5	2:30-3:20	S. New
6	9:30-10:20	D. Park
7	8:30-9:20	C. Hewitt
9	9:30-10:20	F. Vinette
10	8:30-9:20	D. Park
11	11:30-12:20	C. Hewitt

Pages: This test contains 10 pages, including this page and two blank pages at the end for rough work.

Instructions: Write your name, signature and ID number, and circle your section, at the top of this page. Answer all questions, and provide full explanations. No calculators are allowed.

Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
Total	/70

[2] **1:** (a) Find $\lim_{x \to 1} \dfrac{x^2 - 1}{\sqrt{x} - 1}$.

[3] (b) Let $f(x) = (x-1)^2$. Use the definition of the derivative to show that $f'(4) = 6$.

[5] (c) Let $f(x) = \dfrac{6}{x}$. Use the definition of the limit to show that $\lim_{x \to 2} f(x) = 3$.

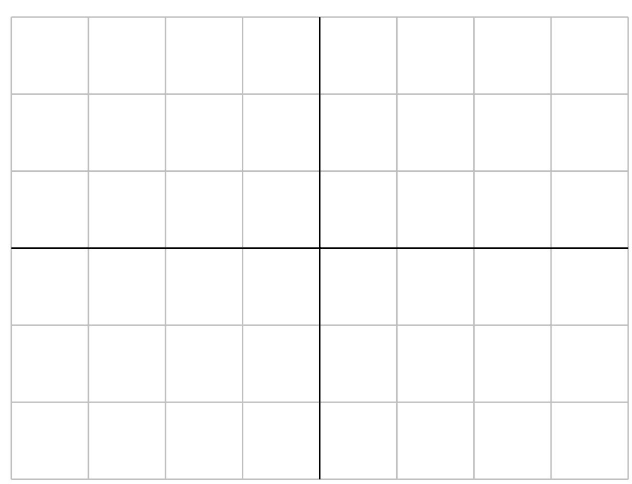
[5] **2:** (a) Approximate the value of $\sqrt{5}$ by finding the approximations x_2 and x_3 when Newton's Method is applied to the function $f(x) = x^2 - 5$ starting with $x_1 = 1$.

[5] (b) Let $y = g(x)$ be defined implicitly by the equation $y^3 + x^2 y = x + 3y^2$ with $g(2) = 1$. Use implicit differentiation to find $g'(2)$, then use the linearization of $g(x)$ at $x = 2$ to approximate the value of $g\left(\frac{5}{3}\right)$.

3: Let $f(x) = \ln\left(\frac{x^2+1}{4}\right)$.

[5] (a) Determine where each of $f(x)$, $f'(x)$ and $f''(x)$ is positive, negative, and zero.

[5] (b) Sketch the curve $y = f(x)$ showing all x and y-intercepts, all local maxima and minima, and all points of inflection. (Note that $\ln 2 \cong 0.7$).



[5] **4:** (a) Let L be a line with negative slope which passes through the point $(2,1)$. Find the minimum possible area for the triangle bounded by L and the x and y-axes.

[5] (b) Let $a=(0,0)$, $b=(3,0)$ and $c=(2,y)$. Let θ be the angle at c in the triangle abc. The point c moves downwards with $y'=-1$. Find θ' when $y=1$. (Note that y' and θ' denote derivatives with respect to time t).

[3] **5:** (a) Find $\displaystyle\int_0^{\pi/6} \frac{\cos x \; dx}{\sqrt{1+6\sin x}}$.

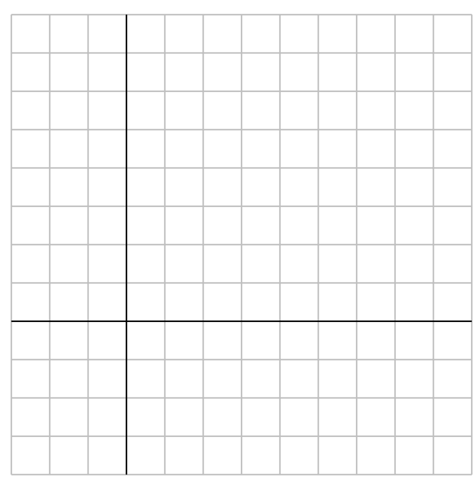
[3] (b) Let $g(x) = \displaystyle\int_1^{\sqrt{x}} \sqrt{5+t^2}\,dt$. Find $g'(4)$.

[4] (c) Evaluate $\displaystyle\int_{-1}^{2} x^2+1 \; dx$ by finding a limit of Riemann sums using the right endpoints of n equal-sized subintervals.

Recall that $\sum_{i=1}^{n} 1 = n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n\left(n+\frac{1}{2}\right)\left(n+1\right)}{3}$, and $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$.

[5] **6:** (a) An object moves along the x-axis with acceleration at time t given by $a(t) = \dfrac{2}{\sqrt{t+1}} - 1$ for $0 \leq t \leq 8$. Given that $x(0) = v(0) = 0$, find $x(8)$.

[5] (b) Find the area of the region bounded by the curves $y = x(x-4)$ and $y = \dfrac{2x}{x-3}$.



7: Suppose that $f(x)$ is defined for all x in an open interval I with $a \in I$.

[4] (a) Prove the Decreasing Test: if $f'(x) < 0$ for all $x \in I$ then f is decreasing in I.

[6] (b) Prove Fermat's Theorem: if $f'(a)$ exists and f has a local maximum or minimum at $x = a$, then $f'(a) = 0$.

This page is for rough work. It will not be marked.

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