MATH 137 Calculus 1, Assignment 9 — Due Fri Nov 23

1: Let T be the triangle with vertices at $a = (-1, 0)$, $b = (1, 0)$ and $c = (0, 2)$.

(a) A point p lies inside T along the y-axis. Find the smallest possible value for the sum $|p - a| + |p - b| + |p - c|$ (where $|p - a|$ denotes the distance between p and a).

(b) A smaller triangle S, with its lower vertex at $(0, 0)$ and its upper edge parallel to the x-axis, is inscribed in T. Determine the minimum possible perimeter of S.

2: Let $a = (1, 5)$ and $b = (2, 2)$ and let $c = (x, 0)$ with $x \geq \frac{8}{3}$ be a point along the x-axis.

(a) Find the maximum possible angle at c in the triangle abc.

(b) Find the maximum possible length for the shadow along the y-axis cast by the line segment ab when a light is placed at point c.

3: (a) Find the maximum possible capacity of a conical cup which is made from a circular piece of paper, of radius 3, with a slit along a radius.

(b) Find the volume of the largest cone which can be inscribed in a sphere of radius $\frac{3}{2}$ m.

4: (a) Let $f(x) = x^3 - 3x + 1$ and let $x_1 = 0$. Apply Newton's method to find the approximations x_2 and x_3 to one of the roots of f. Sketch the graph of f and indicate which root is being approximated.

(b) Let $f(x) = x^3 - 4x$. Find $x_1 > 0$ such that when Newton's method is applied, we obtain $x_n = (-1)^{n+1} x_1$. Draw a sketch which explains the situation.

5: Use MAPLE to apply Newton's Method on a suitable function to find the approximate distance (accurate to about 10 decimal places) from the point $(1, 2)$ to the curve $y = \ln x$.

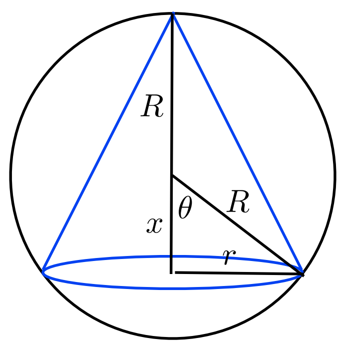
Hints and Comments

2: (a) Let θ be the angle at c in the triangle abc. Let α be the angle between line ac and the horizontal, and let β be the angle between line bc and the horizontal. With the help of a diagram, find expressions for $\tan\alpha$ and $\tan\beta$ in terns of x, and note that $\theta = \alpha - \beta$ to obtain a formula for θ in terms of x.

(b) Say the shadow cast by the point a is at position $(0, u)$ and the shadow cast by the point b is at position $(0, v)$. With the help of a diagram, find expressions for u and v in terms of x. The length of the shadow cast by the line segment ab is then given by $L = u-v$.

3: (a) Let r be the radius of the circular rim of the conical cup (that is the radius of the base of the cone) and let h be the height of the cup (that is the depth of the water in the cup when it is filled to capacity). By visualizing the way in which the cup is formed from the circular piece of paper of radius 3, you should convince yourself that $r^2 + h^2 = 3^2$. The volume of the conical cup (that is the volume of the water that it contains) is given by $V = \frac{1}{3}\pi r^2 h$. Express V in terms of a single variable, say r.

(b) Let $R = \frac{3}{2}$ and let r, x and θ be as shown. Express the volume of the cone in terms of your favourite variable (r, x, θ or some other variable of your choice).



4: (b) Find the recursion formula for x_{n+1} in terms of x_n and then solve $x_{n+1} = -x_n$ for x_n.

5: Either let $f(x)$ be the distance, or the square of the distance, between the point $(1, 2)$ and the point $(x, \ln x)$ (an arbitrary point on the curve $y = \ln x$). The distance from $(1, 2)$ to the curve $y = \ln x$, is the minimum possible distance from $(1, 2)$ to any point $(x, \ln x)$ on the curve, so we need to minimize $f(x)$. To minimize $f(x)$ we wish to solve $f'(x) = 0$. Either let $g(x) = f'(x)$, or let $g(x)$ be the numerator of $f'(x)$, then apply Newton's Method to find the (unique) root of $g(x)$. You are asked to instruct MAPLE to perform Newton's Method. You should do this using a

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for ... while ... do
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loop. You will find that not many steps are required to reach 10 decimal place accuracy.