MATH 137 Calculus 1, Assignment 5 Due Fri Oct 19

1: (a) Let $f(x)=\sqrt{5-x^2}$. Using the definition of the derivative as a limit, find $f'(2)$ and then find the equation of the tangent line to the curve $y=f(x)$ at the point where $x=2$.

(b) Let $f(x)=x^3+x-1$. Find $f'(1)$ using the definition of the derivative. and then find the equation of the tangent line to $y=f(x)$ at the point where $x=1$.

2: (a) Let $f(x)=\dfrac{1}{x}$. Find the derivative $f'(x)$ using the definition of the derivative.

(b) Let $f(x)=x^{1/3}$. Use the definition of the derivative to show that $f'(0)$ does not exists and to show that $f'(x)=\frac{1}{3}x^{-2/3}$ for $x\neq 0$.

3: (a) Let $f(x)=\dfrac{x^2-5}{x-2}$. Find the equation of the tangent line to $y=f(x)$ at $(3,4)$.

(b) Let $f(x)=\dfrac{\sqrt{x}}{e^x}$. Find all the values of x where the tangent line to $y=f(x)$ is horizontal.

(c) Find the equations of the two lines which are tangent to the curve $y=\dfrac{x^2}{x-1}$ and which pass through the point $(2,0)$.

4: (a) Suppose that $f\left(\frac{1}{4}\right)=8$ and that $f'(x)=\dfrac{1+x\,f(x)}{\sqrt{x}}$. Find $f''\left(\frac{1}{4}\right)$.

(b) Suppose that $f(1)=4$, $f'(1)=2$ and $f''(1)=6$, and let $g(x)=\dfrac{x\,f(x)}{1+x}$. Find $g''(1)$.

(c) Let $f(x)=x^2e^x$. Find the $f^{(n)}(x)$, the n^{th} derivative of $f(x)$, in terms of n and x.

5: (Not to be handed in) If you have note done so already, then you should obtain a copy of MAPLE (or make sure that you have access to MAPLE on the 3rd floor computers) and you should complete the MAPLE tutorial which is posted on the LEARN site. Note that MAPLE will not be required for the midterm test, but it will be used in some upcoming assignments.

Hints and Comments

2: (b) Use the formula $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

3: (c) Find the equation of the line L_a which is tangent to the curve $y = f(x)$ at the point $\big(a, f(a)\big)$, then find the values of a for which the line L_a passes through the point $(2,0)$.

4: (c) Find the first few derivatives of $f(x)$, guess a formula for the n^{th} derivative $f^{(n)}(x)$, then prove your formula using induction.