MATH 137 Calculus 1, Assignment 4 Due Fri Oct 12

1: Evaluate the following limits, if they exist.

(a) $\displaystyle\lim_{x\to-\infty}\frac{(2x-1)^3}{(4x+2)(x-1)^2}$

(b) $\displaystyle\lim_{x\to\infty}\frac{x\left(\sqrt{x}+1\right)}{x^2+3x+2}$

(c) $\displaystyle\lim_{x\to-\infty}\frac{3x+2}{\sqrt{4x^2+x+1}}$

(d) $\displaystyle\lim_{x\to\infty}\left(\sqrt{x^2+6x}-x\right)$

2: Evaluate the following limits if they exist.

(a) $\displaystyle\lim_{x\to\frac{\pi}{2}} e^{\sin x}$

(b) $\displaystyle\lim_{x\to\infty}\ \tan^{-1}(\ln x)$

(c) $\displaystyle\lim_{x\to\infty}\frac{\cos x}{x^2+1}$

(d) $\displaystyle\lim_{x\to 1^+}\big(2\log(x-1)-\log(x^2-1)\big)$

3: (a) Sketch the graph of $y=f(x)$ and find all points where f is continuous, where

$$f(x)=\begin{cases}\pi\, e^x & x\le 0\\ 2\,\cos^{-1}x & 0<x\le 1\\ \ln x & 1<x\end{cases}$$

(b) Find the values of a and b such that $f(x)$ is continuous for all x, where

$$f(x)=\begin{cases}\dfrac{x^2+ax+b}{x-1} & x<1\\ ax+b & x\ge 1\end{cases}$$

4: Let $f(x)=\dfrac{x^2-1}{2x^2-x^3}$ and let $g(x)=e^{f(x)}$.

(a) Find $\displaystyle\lim_{x\to-\infty}g(x)$, $\displaystyle\lim_{x\to-1}g(x)$, $\displaystyle\lim_{x\to0}g(x)$, $\displaystyle\lim_{x\to1}g(x)$, $\displaystyle\lim_{x\to2^-}g(x)$, $\displaystyle\lim_{x\to2^+}g(x)$ and $\displaystyle\lim_{x\to\infty}g(x)$.

(b) Sketch the graph of $y=g(x)$.

(c) Sketch the graph of $y=1/g(x)$.

5: (a) Show that there exist (at least) 3 distinct values of x such that $8x^3=6x+1$.

(b) Let $f(x)$ be continuous on $[0,2]$ with $f(0)=f(2)$. Show that $f(x)=f(x+1)$ for some $x\in[0,1]$.

(c) Let $f(x)$ be $1:1$ and continuous on the interval $[a,b]$ with $f(a)<f(b)$. Show that the range of f is the interval $\big[f(a),f(b)\big]$.

Hints and Comments

1: (c) When $x < 0$ we have $\sqrt{x^2} = |x| = -x$.

(d) Rationalize the numerator

3: (b) If $\lim_{x \to a} g(x) = 0$ then in order for $\lim_{x \to a} \frac{f(x)}{g(x)}$ to exist and be finite, we must have $\lim_{x \to a} f(x) = 0$.

4: (b) The limits that you have found in part (a) provide enough information to obtain a fairly accurate sketch of the graph of $y = g(x)$.

5: (a) Apply the Intermediate Value Theorem (3 times) to the function $f(x) = 8x^3 - 6x - 1$.

(b) Apply the Intermediate Value Theorem to a suitably chosen function.

(c) Use the Intermediate Value Theorem both to show that $\big[f(a), f(b)\big] \subseteq \text{Range}(f)$ and to show that that $\text{Range}(f) \subseteq \big[f(a), f(b)\big]$.