MATH 137 Calculus 1, Assignment 3 Due Fri Oct 5

1: Evaluate the following limits, if they exist.

(a) $\displaystyle\lim_{x\to 2}\sqrt{\frac{x^2+5x-14}{x^2-4}}$

(b) $\displaystyle\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x-1}$

(c) $\displaystyle\lim_{x\to 4}\frac{x-\sqrt{x}-2}{x-4}$

2: Evaluate the following limits, if they exist.

(a) $\displaystyle\lim_{x\to 1^-}\frac{x^2-3x+2}{x^3+x^2-5x+3}$

(b) $\displaystyle\lim_{x\to 2^-}\frac{x-2}{\sqrt{4-x^2}}$

(c) $\displaystyle\lim_{x\to 3^-}\frac{|6+x-x^2|}{1-|4-x|}$

3: Evaluate the following limits, if they exist.

(a) $\displaystyle\lim_{x\to 0^+} x\sqrt{1+\tfrac{1}{x}}$

(b) $\displaystyle\lim_{x\to 0} x^2\left(1+\sin\tfrac{1}{x}\right)$

(c) $\displaystyle\lim_{x\to 0}\,(x^2+1)\cos\tfrac{1}{x}$

4: (a) Use the definition of the limit to show that $\displaystyle\lim_{x\to -2}(3x+2)=-4$.

(b) Use the definition of the limit to show that $\displaystyle\lim_{x\to -1}\frac{x+1}{x^2-1}=-\tfrac{1}{2}$.

(c) Let $a>0$ and let $0<\epsilon<\frac{1}{a}$. Find the *largest* value of $\delta>0$ with the property that for all x with $0<|x-a|<\delta$ we have $\left|\frac{1}{x}-\frac{1}{a}\right|<\epsilon$.

5: (a) Use the definition of the limit to show that for all $a>0$ we have $\displaystyle\lim_{x\to a}\sqrt{x}=\sqrt{a}$.

(b) Use the definition of the limit to show that if the limit exists then it is unique, that is if $\displaystyle\lim_{x\to a}f(x)=L$ and $\displaystyle\lim_{x\to a}f(x)=M$ then $L=M$.

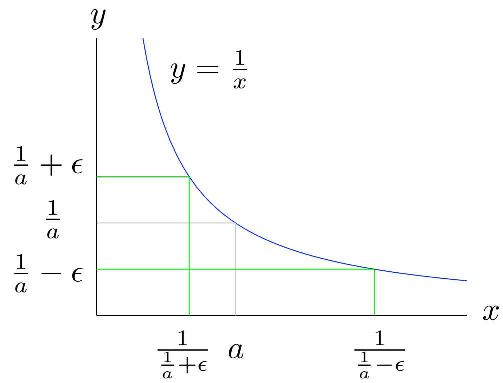
Hints and Comments

1: (b,c) Try rationalizing the numerator.

2: (c) Note that when $x < 3$ we have $4 - x > 0$ so $|4 - x| = 4 - x$. In a similar way, consider the absolute values of the factors of $6 + x - x^2$.

3: The Squeeze Theorem is only useful for one of the three parts.

4: (c) Consider the following diagram.



5: (a) Note that $\sqrt{x} - \sqrt{a} = \dfrac{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}{\sqrt{x} + \sqrt{a}}$.

(b) By the Triangle Inequality, we have

$$|L - M| = \Big|\big(L - f(x)\big) + \big(f(x) - L\big)\Big| \leq \big|L - f(x)\big| + \big|f(x) - M\big| .$$