MATH 137 Calculus 1, Assignment 11 Not to hand in

1: Evaluate the following definite integrals.

(a) $\displaystyle\int_{-2}^{4} x^3 - x^2 - 3x + 1 \, dx$

(b) $\displaystyle\int_{\pi/6}^{\pi/2} \cos x - \sqrt{3}\, \sin x \, dx$

(c) $\displaystyle\int_{1}^{4} \frac{(x-1)(x-2)}{\sqrt{x}} \, dx$

2: Evaluate the following definite integrals.

(a) $\displaystyle\int_{0}^{\ln 3} \frac{e^x \, dx}{1+e^x}$

(b) $\displaystyle\int_{0}^{3} \frac{x^2 \, dx}{(x+1)^{3/2}}$

(c) $\displaystyle\int_{0}^{\pi/3} \frac{\sin^3 x}{\cos^2 x} \, dx$

3: Evaluate the following definite integrals.

(a) $\displaystyle\int_{0}^{2} \frac{x^3 \, dx}{\sqrt{2x^2+1}}$

(b) $\displaystyle\int_{1}^{3} \frac{\sqrt{x}\, dx}{x+1}$

(c) $\displaystyle\int_{0}^{\pi/4} \tan^3 x \, dx$

4: (a) Find the area of the region which is bounded by the curves $y^2 = 2x$ and $y = \dfrac{x}{x-3}$.

(b) Find the area of the region bounded by $y = \sin x$ and $y = 1 - \frac{1}{\sqrt{3}} \sin 2x$ with $0 \le x \le 2\pi$.

5: (a) Find $\displaystyle\int \tan^{-1} x \, dx$

(b) Find $\displaystyle\int \sqrt{1-x^2} \, dx$

Hints and Comments

1: These integrals can be found by inspection.

2: Each of these integrals can be found using a suitable substitution.

3: These can all be found using suitable substitutions.

4: (a),(b) It helps to sketch the given regions. Your sketch should be sufficiently accurate that you can determine the exact coordinates of the points of intersection of the bounding curves. Alternatively, you can determine the points of intersection algebraically.

5: (a) Use trial and error to find an antiderivative. As a first attempt, let $f(x) = x\tan^{-1}x$. and find $f'(x)$. Although, $f'(x) \neq \tan^{-1}x$, you should be able to find another function $g(x)$ so that $f'(x) - g'(x) = \tan^{-1}x$.

(b) Let $f(t) = \sqrt{1-t^2}$ and let $F(x) = \int_0^x f(t)\,dt$. By the FTC we know that $F(x)$ is an antiderivative of $f(x) = \sqrt{1-x^2}$. Note that for $0 \leq x \leq 1$, $F(x)$ is equal to the area under the curve $y = f(t)$ with $0 \leq t \leq x$. Since the graph $y = f(t)$ is a semicircle, you can use a bit of geometry to work out an explicit formula for $F(x)$.