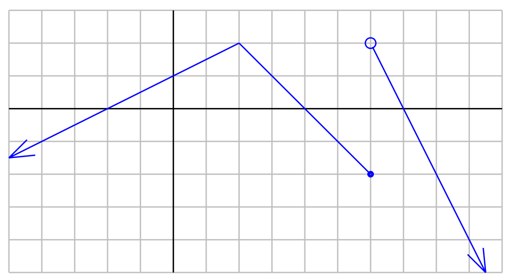
MATH 137 Calculus 1, Assignment 10 Not to hand in

1: (a) Find the function $f(x)$, defined for $x > 0$, such that $f'(x) = \dfrac{\sqrt{x}+1}{x^2}$ with $f(1) = 0$.

(b) Find the function $f(x)$ such that $f''(x) = e^x - \sin x$ with $f(0) = 2$ and $f'(0) = 1$.

(c) An object moves along the x-axis with $a(t) = |t-1| - 1$. Given that $x(t)$ and $v(t)$ are both continuous and $x(0) = 0$ and $v(2) = 0$, find $x(2)$.

2: Let $g(x) = \displaystyle\int_0^x f(t)\,dt$ where $f(t)$ is the function whose graph is shown below. Sketch the graph of $y = g(x)$ showing all intercepts, all local maxima and minima, and all points of inflection.



3: (a) Let $f(x) = \dfrac{8x}{2^{3x}}$. Approximate the integral $\displaystyle\int_0^2 f(x)\,dx$ using the Riemann sum for $f(x)$ which uses the right endpoints of 6 equal-sized subintervals.

(b) Let $f(x) = \dfrac{1}{x}$. Approximate the integral $\displaystyle\int_{1/5}^{13/5} f(x)\,dx$ using the Riemann sum for $f(x)$ which uses the midpoints of 6 equal-sized subintervals.

4: (a) Evaluate $\displaystyle\int_1^3 x^3 - 3x\ dx$ by finding a limit of Riemann sums.

(b) Evaluate $\displaystyle\int_0^1 e^x\ dx$ by finding a limit of Riemann sums.

5: (a) Find $g'(1)$ where $g(x) = \displaystyle\int_{3x-3}^{x^2+1} \sqrt{1+t^3}\,dt$.

(b) Find $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n+i}$.

Hints and Comments

1: (a) By inspection, find a function $g(x)$ with $g'(x) = \dfrac{\sqrt{x}+1}{x^2}$. To get $f'(x) = g'(x)$ for all $x > 0$, you need $f(x) = g(x) + c$ for some constant c. Use the condition that $f(1) = 0$ to find the constant c.

(c) It might help to express $a(t)$ in the piecewise form

$$a(t) = \begin{cases} -t\,, & \text{if } t \le 1 \\ t-2, & \text{if } t \ge 1\,. \end{cases}$$

2: The graph of $y = g(x)$ can be found by interpreting the integral as a signed area. Alternatively, you can find an explicit formula for $g(x)$ by first obtaining an explicit piecewise formula for $f(x)$, which you can determine from its graph.

3: (b) We remark that, by the FTC, the exact value of the given integral is

$$\int_{1/5}^{13/5} \frac{dx}{x} = \Big[\ln x\Big]_{1/5}^{13/5} = \ln\tfrac{13}{5} - \ln\tfrac{1}{5} = \ln 13\,,$$

and so the approximation of the integral by Riemann sums can be thought of as a numerical approximation of the value $\ln 13$.

4: (a) You will need the formulas

$$\sum_{i=1}^{n} 1 = n\,,\ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}\,,\ \sum_{i=1}^{n} i^2 = \frac{n\big(n+\frac{1}{2}\big)\big(n+1\big)}{3}\,,\ \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}\,.$$

(b) The Riemann sum that you obtain should be geometric. Recall that the sum of a geometric series is given by the formula

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(r^n-1)}{r-1}\,.$$

To find the limit of the Riemann sums, you can make use of l'Hôpital's Rule (strictly speaking, in order to use l'Hôpital's Rule, you should first replace the discreet variable n by a continuous variable x).

5: (a) Let $u(x) = x^2+1$, $v(x) = 3x-3$, $f(t) = \sqrt{1+t^3}$ and $F(u) = \int_0^u f(t)\,dt$. Show that $g(x) = F\big(u(x)\big) - F\big(v(x)\big)$ and then use the Chain Rule and the FTC to find $g'(x)$.

(b) Find a function $f(x)$ and an interval $[a,b]$ such that the given sum is equal to a Riemann sum for $f(x)$ on $[a,b]$.