Last Name (print): ______________________

First Name (print): ______________________

Signature: ______________________

ID Number: ______________________

Section (circle): 1 2 3 4 5 6 7 8 9

MATH 135, Algebra for Honours Mathematics

Faculty of Mathematics, University of Waterloo

Term Test 2, Fall Term 2009

Date: Monday, November 16

Time: 7:00 pm-8:50 pm

Section	Time	Instructor
1	10:30-11:20	C. Hewitt
2	12:30-1:20	E. Teske
3	9:30-10:20	S. Furino
4	10:30-11:20	E. Teske
5	11:30-12:20	S. New
6	1:30-2:20	Y.-R. Liu
7	2:30-3:20	R. Moosa
8	12:30-1:20	J. Koeller
9	8:30-9:20	J. Koeller

Pages: This test contains 7 pages, including this cover sheet and a page at the end for rough work.

Instructions: Write your name, signature and ID number, and circle your section, at the top of this page. Answer all questions, and provide **full explanations**. If you need more space to show your work, then use the back of the previous page.

Aids: Only faculty approved calculators are allowed.

Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

[5] **1:** (a) Find all pairs of integers x and y such that $72x - 51y = 24$.

[2] (b) Find all integers c with $0 \leq c \leq 30$ for which there exist integers x and y such that $35x + 56y = c$.

[3] (c) Find the number of pairs of positive integers x and y such that $12x + 18y = 300$.

[3] **2:** (a) Let $a = 10!$ and $b = 60^3$. Find the prime factorizations of $\gcd(a, b)$ and $\text{lcm}(\text{a}, \text{b})$.

[3] (b) Determine the number of positive integers n such that $n \big| 36000$ and $36000 \big| n^2$.

[3] (c) Show that for all positive integers a and b, if $a^3 \big| b^2$ then $a \big| b$.

[1] (d) Show that there exist positive integers a and b such that $a^2 \big| b^3$ but $a \not\big| b$.

[2] **3:** (a) Find the smallest integer n with $n \geq 100$ such that $n \equiv 12 \pmod{17}$.

[2] (b) If a clock now reads 7:00 pm, then what time did it read 500 hours ago?

[3] (c) Let $n = 4,001,005,003,002$. Find all primes p with $1 < p < 12$ such that $p \big| n$.

[3] (d) Show that if $n \equiv 4 \pmod{7}$ then n is not equal to the sum of two cubes.

[2] **4:** (a) Define what it means for an integer p to be prime.

[3] (b) State Fermat's Little Theorem.

[5] (c) Prove Euclid's Theorem, which states that there are infinitely many primes.

[5] **5:** (a) Find every element $x \in \mathbf{Z}_{175}$ such that $[77]x = [84]$.

[5] (b) Find the remainder when $50^{50^{50}}$ is divided by 13.

This page may be used for rough work. It will not be marked.