Name (print): ______________________

Signature: ______________________

ID Number: ______________________

Section (circle): 1 2 3 4 5 6 7 8 9

MATH 135, Algebra for Honours Mathematics

Faculty of Mathematics, University of Waterloo

Term Test 1, Fall Term 2009

Date: Monday, October 19

Time: 7:00 pm-8:50 pm

Section	Time	Instructor
1	10:30-11:20	C. Hewitt
2	12:30-1:20	E. Teske
3	9:30-10:20	S. Furino
4	10:30-11:20	E. Teske
5	11:30-12:20	S. New
6	1:30-2:20	Y.-R. Liu
7	2:30-3:20	R. Moosa
8	12:30-1:20	J. Koeller
9	8:30-9:20	J. Koeller

Pages: This test contains 8 pages, including this cover sheet and a page at the end for rough work.

Instructions: Write your name, signature and ID number, and circle your section, at the top of this page. Answer all questions, and provide full explanations. If you need more space to show your work, then use the back of the previous page.

Aids: Only faculty approved calculators are allowed.

Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

1: Recall that the symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$ are alternate notations for the connectives NOT, AND, OR, $\Longrightarrow$, and $\Longleftrightarrow$, respectively.

[4] (a) Determine whether $P \leftrightarrow (Q \rightarrow \neg P)$ is equivalent to $\neg(P \rightarrow Q)$.

[3] (b) Express the statement "x is the greatest integer such that $2x \leq y$", taking the universe of discourse to be **Z**, and using only symbols from the following list:

$$\neg\ ,\ \wedge\ ,\ \vee\ ,\ \rightarrow\ ,\ \leftrightarrow\ ,\ (\ ,\ )\ ,\ \forall\ ,\ \exists\ ,\ 0\ ,\ 1\ ,+\ ,\ \times\ ,\ =\ ,\ <\ ,\ \leq\ ,\ x\ ,\ y\ ,\ z$$

[3] (c) Determine whether the statement "$\forall x\ x \leq x \times x$" is true when the universe of discourse is **Z** and whether it is true when the universe of discourse is **R**.

[5] **2:** (a) Let $a_0 = 0$ and $a_1 = 1$, and for $n \geq 2$ let $a_n = 5a_{n-1} - 6a_{n-2}$. Show that $a_n = 3^n - 2^n$ for all $n \geq 0$.

[5] **2:** (b) Show that $\sum_{i=0}^{n}(-1)^{i}\, i^2 = (-1)^n \frac{n\,(n+1)}{2}$ for all $n \geq 0$.

[5] **3:** (a) Find the term involving x^1 in the expansion of $\left(x^2 + \frac{1}{2x}\right)^8$.

[5] (b) Evaluate the sum $\displaystyle\sum_{i=0}^{n} \binom{2n+1}{i}$ for all $n \geq 0$. (Prove that your answer is correct).

[2] **4:** (a) Define the statement "a divides b", for integers a and b.

[3] (b) State the Division Algorithm.

[5] (c) Prove Proposition 2.21 from the text, which states that for all integers a, b, q and r, if $a = qb + r$ then $\gcd(a, b) = \gcd(b, r)$.

[5] **5:** (a) Let $a = 231$ and $b = 182$. Find integers s and t such that $as+bt = d$, where $d = \gcd(a, b)$.

[5] (b) Prove that for all integers a, b and c, if $a \big| c$ and $b \big| c$ and $\gcd(a, b) = d$ then $ab \big| cd$.

This page may be used for rough work. It will not be marked.