MATH 135 Algebra, Assignment 9 Not to be handed in

1: Solve the following pairs of congruences.

(a) $x \equiv 5 \pmod 7$
$x \equiv 8 \pmod{15}$

(b) $x \equiv 45 \pmod{84}$
$x \equiv 61 \pmod{115}$

2: Solve the following pairs of congruences.

(a) $15x \equiv 4 \pmod{26}$
$24x \equiv 6 \pmod{63}$

(b) $2x^3 \equiv 7 \pmod 9$
$x^2 \equiv x + 6 \pmod{35}$

3: Chinese generals used to count their troops by telling them to form groups of some size n, and then counting the number of troops left over. Suppose there were 5000 troops before a battle, and after the battle it was found that when the troops formed groups of 5 there was 1 left over, when they formed groups of 7 there were none left over, when they formed groups of 11 there were 6 left over, and when they formed groups of 12 there were 5 left over. How many troops survived the battle?

4: (a) Find $\phi(n)$ for all integers n with $20 \leq n \leq 30$.

(b) Find all positive integers n such that $\phi(n) = 60$.

5: (a) Show that $2^{340} \equiv 1 \pmod{341}$.

(b) Show that $21 \big| (4n^7 + 7n^3 + 10n)$ for all integers n.

(c) Find a positive integer k such that the number 3^k ends with the digits 0001.

(d) Let $n = p^k$ for some positive integer k where p is prime with $p \equiv 3 \pmod 4$. Show that the congruence $x^2 \equiv -1 \pmod n$ has no solution.