MATH 135 Algebra, Assignment 7 Due: Wed Nov 4, 8:30 am

1: (a) Find the smallest non-negative integer x such that $x \equiv 41 \pmod 9$.

(b) Find the integer x which has the smallest absolute value such that $x \equiv 568 \pmod{41}$.

(c) What day of the week will it be 1000 days after a Monday?

(d) What time of day will it be 1000 hours after 5:00 pm?

(e) Exactly what time of day will it be 1 million seconds after 5:00 pm?

2: (a) Find all positive integers m such that $126 \equiv 35 \pmod m$.

(b) Find the remainder when the integer $\sum_{k=1}^{100} k!$ is divided by 13.

(c) Find the remainder when the integer $\dfrac{40!}{2^{20} \cdot 20!}$ is divided by 8.

3: Most recent books are identified by their *International Standard Book Number*, or ISBN, which is a 10-digit number, separated into four blocks. The ISBN for our course text book is 0-13-184868-2. Here the first block of digits, 0, represents the language of the book (English), the second block, 13, represents the publisher (Pearson Prentice Hall), the third block, 184868, is the number assigned to the book by the publisher, and the last block, 2, is the *check digit*. If $a_1, a_2, \cdots, a_{10}$ are the 10-digits of the ISBN then for $1 \leq i \leq 9$ we have $a_i \in \{0, 1, 2, \cdots, 9\}$ while $a_{10} \in \{0, 1, 2, \cdots, 9, X\}$. The check digit a_{10} is used to determine whether an error has been made when an ISBN is copied. It is chosen so that

$$\sum_{i=1}^{10} i\, a_i \equiv 0 \pmod{11}.$$

(a) Determine whether the number 2-14-013862-5 is a valid ISBN.

(b) Determine the value of the digit a such that the number 1-29-14a238-X is a valid ISBN.

(c) When an ISBN was copied, two adjacent digits were interchanged resulting in the number 0-07-286593-4. Determine the original ISBN.

4: (a) Use mathematical induction to show that $5^n \equiv 1 + 4n \pmod{16}$ for all integers $n \geq 0$.

(b) Show that if $n \equiv 3 \pmod 4$ then n is not the sum of two squares.

(c) Show that there is no perfect square whose last three digits are 341.

5: (a) Find all possible pairs of digits (a, b) such that $99 \big| 38a91b$.

(b) Show that it is not possible to rearrange the digits of the number 51328167 to form a perfect square or a perfect cube or any higher perfect power.

(c) Let $n = a_0 + a_1 \cdot 1000 + a_2 \cdot 1000^2 + \cdots + a_l \cdot 1000^l$ where $a_l \neq 0$ and for each i we have $0 \leq a_i < 1000$. Show that for $d = 7$, 11 and 13 we have

$$d \big| n \iff d \big| \big(a_0 - a_1 + a_2 - a_3 + \cdots + (-1)^l a_l\big).$$