MATH 135 Algebra, Assignment 5 Not to be handed in

1: Solve each of the following linear diophantine equations.

(a) $42x + 30y = 24$

(b) $231x + 792y = 513$

(c) $385x - 1183y = 294$

2: (a) Find all non-negative solutions to the diophantine equation $483x + 336y = 9513$.

(b) Find all pairs of integers (x, y) with $x \geq 1000$, $y \leq 1000$ such that $726x - 1578y = 324$.

3: (a) What combinations of 18- and 33-cent stamps can be used to mail a package which requires postage of 6 dollars.

(b) A shopper spends $19.81 to buy some apples which cost 35 cents each and some oranges which cost 56 cents each. What is the minimum number of pieces of fruit that the shopper could have bought.

4: We can solve a pair of linear diophantine equations in three variables by first eliminating one of the variables and solving the resulting equation in the remaining two variables.

(a) Show that there is no solution to the pair of diophantine equations

$$\begin{aligned} 2x + 7y + z &= 45 \\ 3x + 8y + 4z &= 21\,. \end{aligned}$$

(b) Find all solutions to the pair of diophantine equations

$$\begin{aligned} 20x + 12y + 15z &= 85 \\ 18x + 20y + 8z &= 110\,. \end{aligned}$$

5: Let a, b and c be non-zero integers. The **greatest common divisor** $d = \gcd(a, b, c)$ is defined to be the largest positive integer d such that $d \big| a$, $d \big| b$ and $d \big| c$.

(a) Show that $\gcd(a, b, c) = \gcd\big(\gcd(a, b), c\big)$.

(b) Show that for any integer e, the linear diophantine $ax + by + cz = e$ has a solution if and only if $\gcd(a, b, c) \big| e$.

(c) Find all solutions to the linear diophantine equation $42x + 70y + 105z = 63$.