MATH 135 Algebra, Assignment 4 — Due: Wed Oct 14, 8:30 am

1: For each of the following pairs (a, b), find integers q and r with $0 \leq r < |b|$ such that $a = bq + r$.

(a) $a = 753$, $b = 21$

(b) $a = -5124$, $b = 316$

(c) $a = 4137$, $b = -152$

2: For each of the following pairs (a, b), find $\gcd(a, b)$.

(a) $a = 78$, $b = 34$

(b) $a = 456$, $b = 1273$

(c) $a = -1205$, $b = 2501$

3: For each of the following pairs (a, b), find $d = \gcd(a, b)$ then find integers s and t such that $as + bt = d$.

(a) $a = 60$, $b = 35$

(b) $a = 239$, $b = 759$

(c) $a = -5083$, $b = 1656$

4: Prove each of the following statements.

(a) For all integers a, b we have $\gcd(a, b) = \gcd(2a + b, 3a + 2b)$.

(b) For all integers a, b, c with $c > 0$ we have $\gcd(ac, bc) = c\,\gcd(a, b)$.

(c) For all integers a, b, c we have $\gcd(ab, c) = 1$ if and only if $\gcd(a, c) = \gcd(b, c) = 1$.

5: Use long division of polynomials to solve the following problems. (You may find it useful to read part of section 9.1 in the text. An example of long division of polynomials is on page 229, and the statement and proof of the Division Algorithm for Polynomials is on page 230).

(a) Let $a(x) = 4x^5 - x^3 + 2x^2 - 3x + 5$ and $b(x) = 2x^2 + x + 3$. Find polynomials $q(x)$ and $r(x)$ with $\deg(r(x)) < \deg(b(x))$ such that $a(x) = b(x)q(x) + r(x)$.

(b) Let $a(x) = 2x^3 - 3x^2 - 2x + 8$ and $b(x) = x^2 - 3x + 3$. Find polynomials $s(x)$ and $t(x)$ such that $a(x)s(x) + b(x)t(x) = 1$.