MATH 135 Algebra, Assignment 3 Due: Wed Oct 7, 8:30 am

1: (a) Let $a_1 = 1$ and $a_{n+1} = 3a_n + 2$ for $n \geq 1$. Show that $a_n = 2 \cdot 3^{n-1} - 1$ for all $n \geq 1$.

(b) Let $a_1 = 3$ and $a_{n+1} = 2a_n - 1$ for $n \geq 1$. Find a closed form formula for a_n.

(c) Let $a_1 = 2$ and $a_{n+1} = \dfrac{5a_n - 4}{a_n}$ for $n \geq 1$. Show that $1 \leq a_n \leq a_{n+1} \leq 4$ for all $n \geq 1$.

2: (a) Let $a_0 = 0$ and $a_1 = 1$ and for $n \geq 2$ let $a_n = a_{n-1} + 6a_{n-2}$. Show that we have $a_n = \frac{1}{5}\big(3^n - (-2)^n\big)$ for all $n \geq 0$.

(b) Let $a_0 = 1$ and $a_1 = 1$ and for $n \geq 2$ let $a_n = 2a_{n-1} + a_{n-2}$. Show that we have $a_n = \frac{1}{2}\big((1+\sqrt{2})^n + (1-\sqrt{2})^n\big)$ for all $n \geq 0$.

3: (a) Show that $\displaystyle\sum_{i=1}^{n}(2i-1)^2 = \frac{n(2n-1)(2n+1)}{3}$ for all $n \geq 1$.

(b) Find a closed form formula for $\displaystyle\sum_{i=1}^{n}(-1)^i(2i-1)^2$ for $n \geq 1$.

4: (a) Expand $(2x+5)^4$.

(b) Expand $\left(x - \frac{1}{2x}\right)^8$.

(c) Find the term involving x^8 in the expansion of $\left(\frac{x^3}{6} - \frac{12}{x^2}\right)^{11}$.

5: (a) Evaluate $\displaystyle\sum_{i=0}^{n}\binom{n}{i}\frac{1}{2^i}$.

(b) Evaluate $\displaystyle\sum_{i=0}^{n}\binom{2n}{2i}\frac{1}{2^i}$.

(c) Evaluate $\displaystyle\sum_{i=0}^{n}\binom{n+i}{i}\frac{1}{2^i}$.