MATH 135 Algebra, Assignment 2 Due: Wed Sept 30, 8:30 am

Please note that the symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$ are alternate notations for the connectives "NOT", "AND", "OR", "$\Longrightarrow$" and "$\Longleftrightarrow$" respectively.

1: (a) Make a truth table for the statement $(P \leftrightarrow \neg R) \wedge (R \rightarrow Q)$.
(b) Determine whether $(P \wedge \neg Q) \vee (R \leftrightarrow P)$ is equivalent to $Q \leftrightarrow R$.
(c) Determine whether $P \rightarrow (Q \rightarrow R)$ is equivalent to $(P \rightarrow Q) \rightarrow R$.

2: Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. List all of the elements in each of the following sets.
(a) $A = \big\{x \in S \,\big|\, x \text{ is even or } x \text{ is a multiple of } 3\big\}$
(b) $B = \big\{x \in S \,\big|\, \text{if } x \text{ is even then } x \text{ is a multiple of } 3\big\}$
(c) $C = \big\{(x, y) \in S \times S \,\big|\, 3x + 2y = 20 \text{ and } (y < x^2 \text{ if and only if } xy < 8)\big\}$

3: Express each of the statements below symbolically, taking the universe of discourse to be $\mathbf{R}$ and using only symbols from the following list:

$$\neg\ ,\ \wedge\ ,\ \vee\ ,\ \rightarrow\ ,\ \leftrightarrow\ ,\ (\ ,\ )\ ,\ \forall\ ,\ \exists\ ,\ 0\ ,\ 1\ ,+\ ,\ \times\ ,\ =\ ,\ <\ ,\ \leq\ ,\ x\ ,\ y\ ,\ z$$

(a) x is equal to the minimum of y and z.
(b) There is no smallest positive real number.
(c) Every real number has a unique cubed root.

4: For each of the following statements, determine whether it is true when the universe of discourse is $\mathbf{Z}$ and whether it is true when the universe of discourse is $\mathbf{R}$.
(a) $\forall x\, \exists y\, (x = y \times y \ \vee\ y \times y + x = 0)$
(b) $\forall x\, \exists y\, \big(y < x \ \wedge\ \forall z\, (z < x \rightarrow z \leq y)\big)$
(c) $\forall x\, \forall y\, \big((0 < x \ \wedge\ x + y < x \times y) \rightarrow 0 < y\big)$

5: Prove each of the following statements, where the universe of discourse is $\mathbf{R}$.
(a) $\forall x\, \big(\, |x^2 - 4x| \leq x \ \leftrightarrow\ \big(x = 0 \ \vee\ (3 \leq x \ \wedge\ x \leq 5)\big)\big)$
(b) $\forall x\, \forall y\, \big(y^2 = x^3 + x^2 \ \leftrightarrow\ \exists z\, (x = z^2 - 1 \ \wedge\ y = z^3 - z)\big)$